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Manual of Experiments in General Physics

BY

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Preface

This manual is written for the use of sophomore students in the general physics courses at the University of Illinois. The directions are such as apply to the particular forms of apparatus and facilities to be found in this laboratory. The methods of performing experiments, and the designs of special forms of apparatus have been developed under the supervision of Professor A. P. Carman, Director of the Physical Laboratory. In the second edition of this manual, Dr. E. H. Williams, Associate in Physics, has added or has re-written experiments Nos. 4, 6, 10, 18, 19, 20, 22, 25, 30, 48 and 55. The laboratory course runs parallel with the lecture course in general physics. The experiments are performed in groups of three, which are more or less related, but independent so far as the order in which they are performed is concerned. With ten sets of each apparatus, a class of sixty students is provided for in the laboratory at one time, by letting students work together in pairs and having the three related experiments in operation simultaneously. The experiments to be performed during the first semester are described in detail; the directions for those designed for the second semester's work are made less definite, leaving the student some choice in the method of procedure, or giving him a chance to exercise his ingenuity and making him more dependent upon his own knowledge of the underlying principles.

Material for the tables in the appendix has been drawn from a large number of sources, and it would be difficult to give credit to all of these, hence such references have been omitted entirely.

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Introduction.

Laboratory Directions.—In order to facilitate the distribution of apparatus, handling of large classes, etc., the following method of procedure is in use.

The experiments are performed in groups of three, the classes being divided into three groups, and the particular experiment to be performed by each group is posted on the bulletin board.

Experiments are performed in cyclic order; that is, group one will perform the experiments in the order 1-2-3, 4-5-6, etc., group two in the order 2-3-1, 5-6-4, etc., group three in the order 3-1-2, 6-4-5, etc., the order remaining the same throughout the semester.

Students generally work in pairs, the assignment of partners being made at the beginning of each period. Students are required to read carefully the description of the assigned experiment so that they are prepared to perform it without referring to the manual.

The first ten or fifteen minutes of each period are devoted to a discussion of the experiment to be performed. The instructor calls attention to those parts of the experiment which require particular care, and students are given a chance to ask questions concerning those points which they do not understand.

At the beginning of a laboratory period, each student and the partner assigned him for that particular period will be given a key to an apparatus locker, and three blank data sheets on which will be written the number of the experiment assigned, the number of the apparatus locker and the place of working. The laboratory manual gives a list of appa-

ratus needed, and the directions for performing the experiment. This list will be found on the data sheets also. The student will inspect the apparatus in the locker and at the place of working, notifying the instructor at once of missing or damaged apparatus. Students are held responsible for apparatus assigned them. The experiment will be performed in accordance with the laboratory manual, and the results obtained recorded on the data sheets. The measurements taken may be recorded first in pencil on one of the sheets, but must be copied in *ink* on the others. The numerical calculations should be completed in the laboratory so that the instructor may point out any errors.

Apparatus must be returned to the locker, and the place of working be left clean. The students will then return the locker key and data sheets to the instructor, who will stamp them and return the ink copies to be bound in the written report of the experiment.

Alternate periods will be devoted to performing an experiment and writing a report of it, until the student has learned how to write the reports properly. After the first few weeks, three experiments will be performed in successive laboratory periods, and followed by a written report of only one of them. The results of calculations with the data taken, and answers to questions and problems, are however always due at the beginning of the period following that on which the experiment was performed. In any case, each set of experiments will be followed by a written quiz on all three of them.

WRITING OF REPORTS:—The report of each experiment is to be written in ink and bound in one of the flexible covers used for Engineering College reports.

The title page should be properly filled out in free hand lettering.

The data sheet should precede the report and when a curve is plotted it should be placed face to face with the data sheet. The curve must be drawn according to the directions given at the end of this introduction.

In writing up reports the following topics should be treated in logical order:— (a) Object of experiment; (b) Theory; (c) Method; (d) Conclusions and discussion; (e) Questions and problems.

Under "Object of experiment" give a brief and accurate statement of what you think is the object of the experiment.

Under "Theory" give briefly the fundamental theory underlying each experiment and definitions of the physical quantities which have been determined, as well as the equations used in calculating results.

Under "Method" draw (in ink and with instruments) a diagram of the essential parts of the apparatus used whenever this will aid materially in describing the experiment, and give an *abstract* of the process of performing the experiment. Avoid the use of personal pronouns.

Under "Conclusions and Discussion" tell briefly but accurately all that you have learned about the phenomena illustrated by the experiment, and discuss the accuracy of your results, stating the sources and magnitude of errors which may occur and methods of avoiding them, as well as the magnitude of the errors made by neglecting any factors in the computation.

Answer the questions and solve the problems assigned for each experiment, indicating the method of solving and giving the final result.

References:—References will be given to *sections* in the following books:

D.—Duff's Textbook of Physics, Third edition.

G.—Ganot's Physics (Atkinson) Seventeenth edition.

K.—Kimball's College Physics.

W.—Watson's Text book of Physics, Fourth edition.

UNITS:—The magnitude of every physical quantity obtained by measurement or by calculation must include the unit in which this quantity is expressed, and a numeric giving the number of times the unit is contained in the quantity. It is evident that we can have no idea of the magnitude of a physical quantity in general unless it is expressed in terms of a definite unit. Thus to say that a length is two conveys no idea of its size unless we state whether it is two inches or two miles, etc.

On the other hand some physical constants are ratios between two physical quantities of the same kind. For example, the specific gravity of a substance is the ratio of the density of the substance to the density of water, and is expressed as an abstract number without a specific unit being mentioned. Specific heat of a substance is the ratio of the quantity of heat required to raise one gram of the substance one degree Centigrade at a definite temperature to the quantity required to raise one gram of water one degree at the same temperature, and a number is all that is necessary to express this ratio.

SYSTEMS OF UNITS:—D. 149, 150, 154.—All the units of Mechanics may be expressed in terms of three independent units called "fundamental units." All other units are defined by reference to these fundamental units and are called "derived units."

A system of units in which the derived units bear the simplest possible relation to the fundamental units is called "an absolute system."

The three fundamental units in general use are those of length, mass and time. In the C. G. S. absolute system, the centimeter, gram and second are the fundamental units.

In the F. P. S. absolute system, the foot, pound and second are the corresponding fundamental units.

A third system of units, sometimes called the gravitational system, is frequently used. In the absolute systems, the unit of force is defined as the force which gives unit acceleration to the unit mass. The C. G. S. unit of force is called the dyne, the F. P. S. unit the poundal. In the gravitational systems the unit of force is the *weight* of unit mass, or the force with which the earth attracts the unit mass, giving it the acceleration of gravity, "g." Since the gravitational unit of force gives to unit mass in unit time "g" times as much acceleration as does the absolute unit, the gravitational unit of force must equal the absolute unit multiplied by the acceleration of gravity "g". The gram weight is 981 dynes where $g = 981$ cm./sec^2. The pound weight is 32 poundals where $g = 32$ ft./sec^2.

In expressing the result of a measurement or calculation the following directions should be followed:

Be sure to state the unit in which the result is expressed.

Unless otherwise directed use the centimeter, the gram and the second as the unit of length, of mass and of time respectively.

Express all fractions in the decimal form.

To avoid repetition of cyphers, express all large numbers in positive powers of ten, all small numbers in negative powers of ten. For example,

$30{,}000{,}000{,}000 = 3 \times 10^{10}$. $0.0000117 = 11.7 \times 10^{-6}$

Be careful to distinguish between unit mass and unit weight (used as a force). Use the symbols gm. and lb. for gram and pound mass respectively, the symbols gm. wt. and lb. wt. for gram and pound weight respectively.

Where a unit has not been given a name, express it in

terms of fundamental units. For example, in the C. G. S. system, the unit of velocity is one cm. per sec. (cm./sec.); unit acceleration, one cm. per sec. per sec. (cm./sec^2.) etc.

SIGNIFICANT FIGURES:—The accuracy of physical measurements depends amongst other things upon the instruments or apparatus used. For instance in measuring a length, the vernier is correct to 0.01 cm. and the micrometer to 0.001 cm. But it is possible to estimate fractional parts of a scale division on these instruments, so that we can express the length to one more decimal place than that indicated above, although this last figure is somewhat in doubt. It is customary to retain this last figure in the measurement. If however all the doubtful figures are retained in calculations with these measurements, we shall waste considerable time in unprofitable calculations and also give a false idea of the accuracy of the final result.

For instance if we are determining the area of a rectangle and the measurement of the sides gives us 4.3$\underline{5}$ and 11.5$\underline{1}$ respectively, the last figure in each being doubtful, (which is indicated here by underscoring them), the product is 50.$\underline{0}$685, but the underscored 0 and all the figures after it result from multiplication with a doubtful figure, and are therefore doubtful themselves. These figures should therefore be discarded in any further calculations, to save time, the area being written 50.1 sq. cm. If retained they would give the false impression that the area as calculated is correct to the ten thousandth decimal place.

Again if we are determining the density of a body (or mass per unit volume) and find its mass and volume to be 49.$\underline{2}$gm. and 4.35$\underline{2}$cc. respectively, the quotient of these two will give 11.$\underline{3}$05, but the underscored 3 and the following figures are the result of operations with doubtful figures and

are themselves doubtful, so that the density should be written 11.3 gm./cc.

All figures up to and including the first doubtful figure are called *significant* figures.

It is readily seen from these examples that the number of significant figures in the final result is the same as the *least* number of significant figures in any one of the factors.

If the second doubtful figure is less than five it is to be discarded. If it is five or more the first doubtful figure is to be increased by one, as in the first example given.

In computations with logarithms, as many places should be retained in the mantissae as there are significant figures in the data. Thus log 3.1416 = 0.49714. The characteristic is not to be considered in counting the figures.

When the purpose of a zero is simply to fix the decimal place in front of significant figures it is not counted as a significant figure. Thus .0345, 1.03, and 32.0 each have three significant figures.

Where the omission of the second doubtful figure would make a difference as great as, or greater than the experimental error, it should be retained.

Errors:—Since the accuracy to which physical measurements are made depends amongst other things upon the construction of the instruments used, there is always a calculable error in every measurement. Thus a micrometer may indicate a length correctly to 0.001 cm., but by estimating the fractional part of a scale division, the length may be expressed to 0.0001 cm. If the scale divisions are sufficiently large and the eye of the observer sufficiently trained, the measurement may be correct to 0.0001 cm. Usually the readings with the particular micrometer calipers to be used are correct to only 0.0003 cm. or even 0.0005 cm. The possible error in the length from this source alone would

then be 0.0005 cm. and a length X measured with such a degree of accuracy would be recorded $X \pm 0.0005$ cm. It is sometimes desirable to know the effect that such errors have on the final result of computations with these measurements. We shall develop approximate rules for the calculation of such errors.

(*a*)—*Rule for sums and differences.—If the only operations performed with the measurements are addition and subtraction, the maximum possible error in the final result will be the sum of the errors in the measurements.* For instance if two measurements X and Y have errors $\pm a$ and $\pm b$ respectively, their sum will be $(X+Y) \pm (a \pm b)$ and their difference $(X-Y) \pm (a \mp b)$. In each case the maximum error is the sum of a and b, the minimum error their difference.

If now the operations involve multiplication or division, it is convenient to use the percent error rather than the numerical error. A percent error means the error per hundred parts. Thus an error of 2 parts in fifty is an error of 4 parts in 100 or 4%. An error of 0.004 parts in 0.020 is $(0.004/0.020) \times 100$ or 20%. The percent difference between readings of the same quantity is calculated from the mean or average. Thus if two measurements give 49 and 51 units respectively, their mean is $(49+51)/2 = 50$, their difference $51-49=2$, the part difference is 2/50 and the percent difference $(2/50 \times 100) = 4\%$.

The *error from the mean* is sometimes asked for. This is the difference between any one reading and the mean. In the case of two numbers the *percent* error from the mean of the two readings is the same, namely

$$\frac{50-49}{50} \times 100, \text{ or } \frac{51-50}{50} \times 100 = 2\%$$

Or in general, if X and Y be the numbers, the % error

from the mean will be

$$\left| \frac{X-Y}{2} \div \frac{X+Y}{2} \right| \text{ x } 100, \text{ or } \frac{X-Y}{X+Y} \text{ x } 100.$$

(*b*)—*Rule for products and quotients.*—*The maximum possible percent error in a product or in a quotient is the sum of the percent errors in the separate factors provided all of the errors tend to increase or all tend to decrease the product or quotient.* That is, if u, v, and w are respectively the percent errors in X, Y, and Z, then the error in any multiplication or division between these factors (as for instance XY/Z) will be $(u+v+w)$ %, under the condition mentioned. (It should be stated that this rule is only approximate, but that it gives a good idea of the maximum possible per cent error and saves considerable calculation that would be necessary if the numerical errors were carried through the whole calculation.)

To derive this rule let the measurements X and Y have the numerical errors $\pm a$ and $\pm b$ respectively multiplying $(X+a)$ by $(Y+b)$ we get $(X+a)(Y+b)=XY+aY+bX+ab$. a and b are small quantities and their product ab is usually a very small fraction which may be neglected in comparison with aY and bX.

The error in this product is therefore $aX+bY$., for if X and Y were accurate, their product would be XY. The fractional error is therefore the difference between the calculated and true values of the product divided by the true value, or $(aY+bX)/XY=a/X+b/Y$. The percent error is this fraction of 100. The same result is obtained when the negative sign is used with a and b. But 100 a/X is the percent error in X and 100 b/Y is the percent error in Y. Hence the *percent* error in the product is the sum of the *percent* errors in the factors. If now we divide $(X+a)$ by $(Y-b)$, (using opposite signs so that both errors affect

the quotient in the same way) we may write $(X+a)/(Y-b) = (X+a)\ (Y-b)^{-1}$.

Expanding the last term and neglecting products and powers of the errors which become very small, we have $(X+a)\ (1/Y+b/Y^2+\text{etc.}) = X/Y+a/Y+bX/Y^2+\text{etc.}$

The fractional error is therefore $(a/Y+bX/Y^2)/(X/Y) = a/X+b/Y$ and we see as before that the *percent* error in the quotient is the *sum* of the *percent* errors in the numerator and denominator. The same result is obtained when the signs of a and b are interchanged.

(*c*)—*Rule for powers and roots*—If the measurement be raised to any power, the percent error in the power is the percent error in the measurement multiplied by the index of the power, i. e. if squared, the percent error is multiplied by two, for the square of a number is that number multiplied by itself and by rule *b*) the percent error in a product is the sum of the percent errors in the factors. If the root of the measurement is extracted, the same rule applies, for the square root is the measurement raised to the ½ power, etc. Hence the rule for a number raised to a power may be used, and if u be the percent error in X. the percent error in $\sqrt{X}$ will be $u/2$. etc.

Multiplying or dividing the measurement by a constant quantity (in which there is no error of measurement) does not alter the percent error. If K be a constant and X a measurement with a numerical error $\pm a$, then $K\ (X \pm a) = KX \pm Ka$. But Ka is the same percent of KX that a is of X. The same reasoning holds if we multiply by $1/K$, or divide by K.

If on the other hand we divide the constant K by the measurement X with an error of $b\%$, since the error in K is o, the error in the quotient is by our rule $(o+b)\%$ or the same as the error in X.

Percent errors are also useful in judging the relative importance, so far as the final result is concerned, of the errors in the measurements which enter as factors in the calculation of this result. It is evident that a given numerical error is a larger percent of a small measurement than it is of a large measurement. For instance if a micrometer caliper is accurate to 0.0005 cm. then a measurement of a length of 5 mm. made with it will have a possible error of $(0.0005/0.5)\times 100$ or 0.1% whereas a measurement of 10 cm. would have a possible error of $(0.0005/10)\times 100 = 0.005\%$. It is therefore advisable to make the small measurements more carefully than larger ones and with instruments capable of greater precision. But this is not the only thing to be considered, for the factors in a computation do not all have the same importance. Some of the factors enter to a higher degree than others and it should be remembered that a measurement which is cubed has its percent error multiplied by three, etc. Take for example, the volume of a cylinder of length l and radius r. The volume is $\pi r^2 l$, and it is evident that the relative importance of the errors in r and l depends upon their magnitude and the degree to which they are involved. If r and l are equal and are measured with the same instrument, the percent error will be the same in both, but since r is squared while l is not, the error in r has twice as much effect upon the accuracy of the result, as the error in l. If $l=2r$ the percent error in r will be twice the percent error in l and the error introduced into the calculated volume by r^2 will be four times that due to l. If $r=2l$ the percent error in r will be $\frac{1}{2}$ the percent error in l and the percent error in r^2 will equal that in l, so that both are of equal importance in the calculation, etc.

In this way the relative importance of the different factors in a computation may be studied. It is always ad-

visable on beginning an experiment to obtain some idea of the relative magnitudes of the measurements made, (by a preliminary trial if necessary), and the degree to which they are involved in the computation, to determine which should be measured with the greatest care and the most accurate instruments. It must be remembered that the number of significant figures in the result of the computation depends upon the least number of significant figures in any one of the factors.

To avoid useless computation it is advisable to inspect the formulas used in calculating results and to disregard all terms whose omission would introduce negligible errors only. For instance when a shunted galvanometer is used in series with a large resistance, it is well to calculate what percentage of the total resistance is included in the galvanometer and shunt. If their omission introduces a small fraction only of one per cent error in the final result, the term for the shunted galvanometer should be discarded in the calculation.

THE PLOTTING OF CURVES:—The relation between two physical quantities which vary, the one with the other, is often more clearly seen from the curve or graph obtained by plotting on coordinate paper a series of simultaneous observations of the two quantities. Such a curve has other uses besides presenting clearly to the eye the relative variations of the two quantities. For instance we may, by interpolation, determine corresponding values of the two quantities, other than those for which the data have been taken. Again when one or two plotted points deviate very markedly from a smooth curve drawn through the remaining points, we may discard the data for the former as being in error.

Usually such curves are plotted on rectangular coordinate paper having centimeter of half inch spaces divided into tenths. Sometimes however it is advantageous to plot polar

graphs on circular and radial coordinates, or logarithmic curves on specially ruled logarithmic paper. In the following discussion only rectangular coordinates will be used. Two intersecting lines, usually the left hand and lower edge of the ruled space, are chosen as coordinate axes, the horizontal one being called the axis of abscissae, or X, the vertical one the axis of ordinates, or Y. To plot the curve, a measured value of the independent variable is laid off along the axis of abscissae from the intersection of the axes. From this point on the axis the corresponding value of the dependent variable is laid off parallel to the axis of ordinates. The point reached in this way is one point on the curve. This is repeated for each pair of coordinate values, and a smooth curve is drawn through the plotted points.

As a result of many experiments we find that one of the measured quantities varies *directly* as some other, or the *ratio* of their corresponding values is a constant. If we plot corresponding values of such quantities, they will lie along a straight line, generally inclined at an angle to the axes.

The general equation of such a curve is

$$y = a + bx$$

where y is a value of the ordinates, x the corresponding value of the abscissae, a is a constant representing the portion of the axis of ordinates intercepted by the curve, and b a constant denoting the slope of the line. As an example we may take corresponding values of the mass and volume of a substance. The ratio between corresponding values of mass and volume is a constant, the density of the substance, hence in symbols, if m represents mass, v volume and d density,

$$m = dv.$$

If we plot corresponding values of m and v they will lie along a straight line, inclined at an angle with the axis of x whose

tangent is d; for here $y = m$, $x = v$, $b = d$; also, since $a = o$, the line passes through the zero of ordinates.

As a result of other experiments, it is found that one quantity is *inversely* proportional to another, so that the *product* of corresponding values is a constant. If we plot corresponding values of such quantities they will lie along a rectangular hyperbola, having the axes as asymptotes. The equation of such a curve is

$$yx = c$$

where c is a constant. As an example we may take Boyle's law which states that at a constant temperature, the volume of a gas varies inversely as the pressure, or in symbols

$$pv = c$$

Taking $y = p$ and $x = v$, the following corresponding values of p and v are plotted in figure 1, (curve A).

p cm	v cc.	$\frac{1}{v}$
70	50 . 0	. 0200
80	43 . 7	. 0229
90	38 . 9	. 0256
100	35 . 0	. 0285
110	31 . 8	. 0314
120	29 . 1	. 0343

It is evident that one can see more readily whether a series of plotted points lie on a straight line, than whether they lie along an hyperbola or other curve, and also that the former is more easily plotted. It is therefore in general advisable to draw the straight line whenever possible. In many cases one can tell from the trend of the plotted points

what relation exists between the coordinates, and very often it is possible to find some functions of the variables between which a direct ratio exists. On plotting corresponding values of these functions rather than of the variables themselves, they will lie along a straight line. Thus the relation between quantities which are inversely proportional may evidently be expressed as a constant ratio between one of these and the reciprocal of the other, so that

$$y = c(1/x)$$

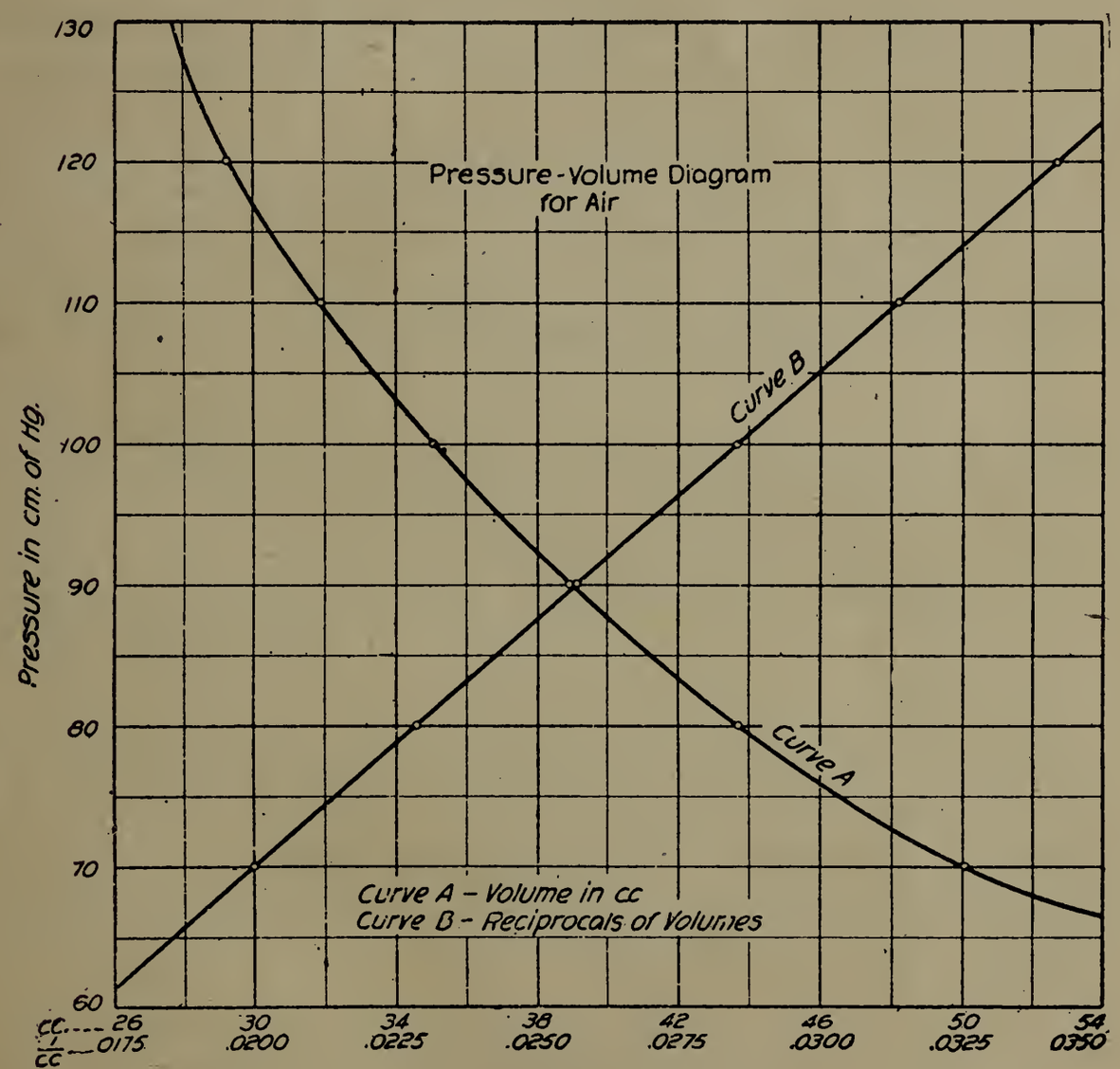


Fig. 1

This is a linear relation between y and $1/x$, and on plotting corresponding values of y and $1/x$ we obtain a straight

line. In figure 1 (curve B) this is shown by plotting corresponding values of p and $1/v$.

As a further example we may take the expression for the strength of an electric current in terms of the deflection which it produces in a tangent galvanometer, namely

$$i = k \tan \phi$$

where i is the current strength, ϕ the deflection and k the reduction factor of the instrument, a constant. If we plot i and ϕ we get a curve. We may judge by inspection that it is a tangent curve, and plot corresponding values of i and $\tan \phi$, which will lie on a straight line. The above equation is then of the form $y = kx$.

As another example of this process we may take the results of the experiment on Joule's law. Here the heating effect of an electric current is proportional to the square of the current strength, or in symbols

$$H = (i^2 R)/J$$

where H is the heat energy, i the current strength, J the mechanical equivalent of heat, a constant, and R the resistance of the circuit, another constant. On plotting H and i, we get a parabola, but corresponding values of H and i^2 when plotted yield a straight line, the equation being of the form $y = (R/J)x$.

Where higher powers of y and x are involved in the equation, it is generally advisable to proceed in a different manner to establish a linear relation in its place, namely, by taking the logarithms of both sides. Taking the general relation

$$y^m = a\ x^n.$$

where m and n are any powers whatever, and passing to logarithms, we have

$$m \log y = \log a + n \log x$$

which is a linear relation between $\log y$ and $\log x$. A simple

example of this is the one already given, namely Boyle's law. This may evidently be written

$$p = c\ v^{-1}$$

or in logarithms

$$log\ p = log\ c - log\ v.$$

If now we plot corresponding values of $log\ p$ and $log\ c - log\ v$, as ordinates and abscissae respectively, we shall find that the plotted points lie along a straight line.

The following rules should be observed in plotting curves:—

Use coordinate paper with millimeter or 1/20 inch spacings on a sheet of the same size as that used in the body of the report.

In order to have the curve facing the data sheet, so that we may refer readily from one to the other, the coordinate paper should be laid with the binding edge on the right hand side while plotting the curve.

Give the curve a title, which where possible, indicates the relation between the two plotted quantities.

Draw two heavy black lines to represent the coordinate axes, and on each of these mark the scale and name of the plotted quantities.

Plot values of the independent variable as abscissae and those of the dependent variable as ordinates.

Make the value of a scale division correspond, as nearly as possible, to the least quantity which you can measure. If this be impossible choose such values for scale divisions, as will make the curve most nearly fill the page with the observations to be plotted.

All lettering and numbers should be placed in such a position that they can be read without turning the paper.

Around each plotted point as a center draw a small circle about one mm. in radius.

Through and between the plotted points draw a smooth regular curve, such that if the points do not all lie on the curve, there will be approximately as many on one side as on the other, alternately.

Any point which deviates markedly from this curve may be assumed to be due to an accidental error and should be discarded.

Experiments in Mechanics

1. DENSITY OF SOLIDS

The object of this experiment is to determine the density of various solids and to become familiar with the methods of using the coarse balance, the vernier calipers and the micrometer calipers.

(References: D. 162. G. 10, 11, 27, 73. K. 69, 159. W. 16-18, 95, 129.)

Apparatus.—Coarse balance and set of weights, vernier calipers, micrometer calipers, cylinders of wood, brass and steel.

Description and Theory.—*The density of a substance is defined as its mass per unit volume.* The mass of the substance is to be found by comparison with known masses on a coarse balance, and its volume to be calculated from its dimensions measured with the vernier and the micrometer calipers.

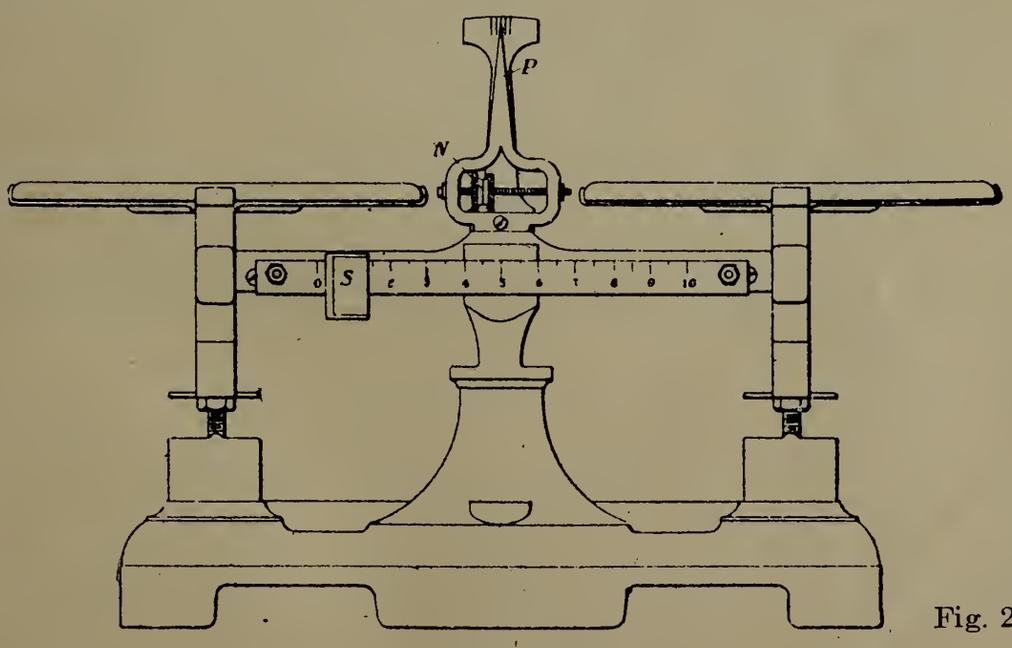


Fig. 2

The coarse balance is so called because it is used where

an accuracy greater than 0.1 gram is not necessary. It is essentially a lever of the first class with equal arms, and when both arms are not equally loaded it is in equilibrium under the action of two equal and opposite moments about the fulcrum. It consists of a beam supported on knife edges at its center, with a pan supported on knife edges at each of its ends. (See Fig. 2.) Two nuts (N) working on a horizontal screw at the center of the beam, serve to bring it into its equilibrium position when there is no load in either pan. A vertical pointer (P) at the center, indicates when this balance position is reached. The unknown mass in one pan is balanced as nearly as possible by known "weights" placed in the other pan. By means of the slider (S) moving along a scale on the front of the beam, the known weights can be adjusted to 0.1 gram.

The vernier calipers consist of a straight scale having a fixed jaw at right angles to it at one end, and a movable jaw bearing a vernier scale. The vernier is a device for measuring fractions of a scale division. The ordinary vernier scales have a space covering $N-1$ divisions of the main scale, divided into N parts, so that each vernier division is

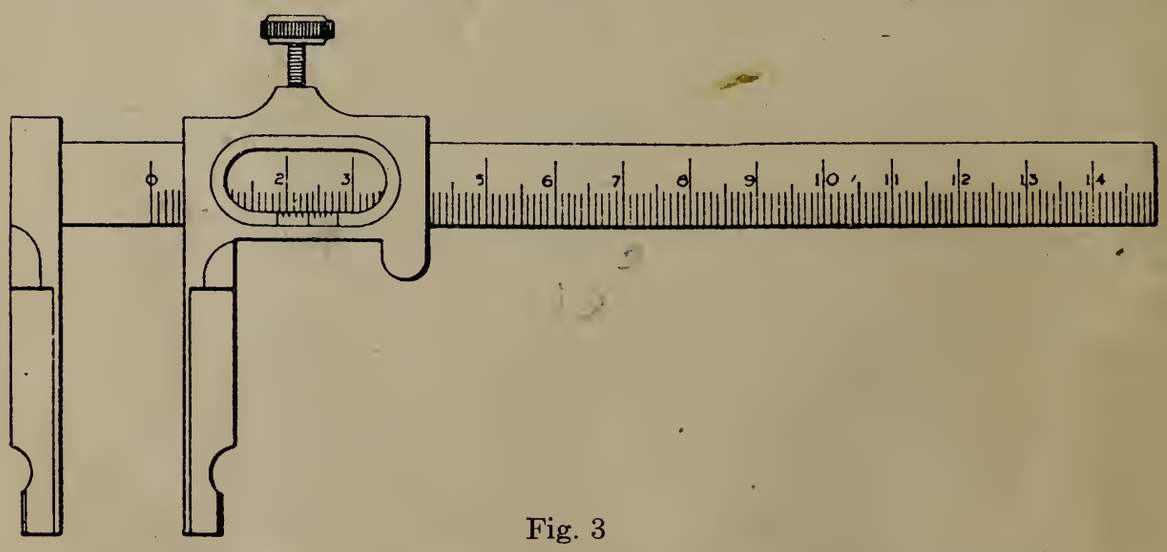


Fig. 3

shorter than a main scale division by one *Nth.* of the latter. $1/N$ is called the vernier fraction. When the two jaws of

the vernier are in contact the zeros of the two scales coincide. If the jaws are separated until the first mark after the zero mark on one scale, coincides with the first mark on the other, the distance between the two zeros (and therefore between the two jaws also) is $1/N$ of a scale division. If separated until the second marks of the two scales coincide, the distance between the jaws is $2/N$ of a scale division; if the third marks coincide $3/N$ of a division, etc., so that when the *Nth.* marks coincide the jaws are separated by a whole division. To determine any distance between the jaws, for instance the length of an object placed between them, we note the whole number of divisions on the main scale up to the zero of the vernier. If this zero does not coincide with a main scale division, the distance is greater than the whole number by a fraction of a division which can be determined by observing the number of the vernier division which coincides (or most nearly coincides) with a main scale division (no matter which one.)

In figure 3, the vernier reads 18.8 mm.

The micrometer calipers consist of a curved or rectangular

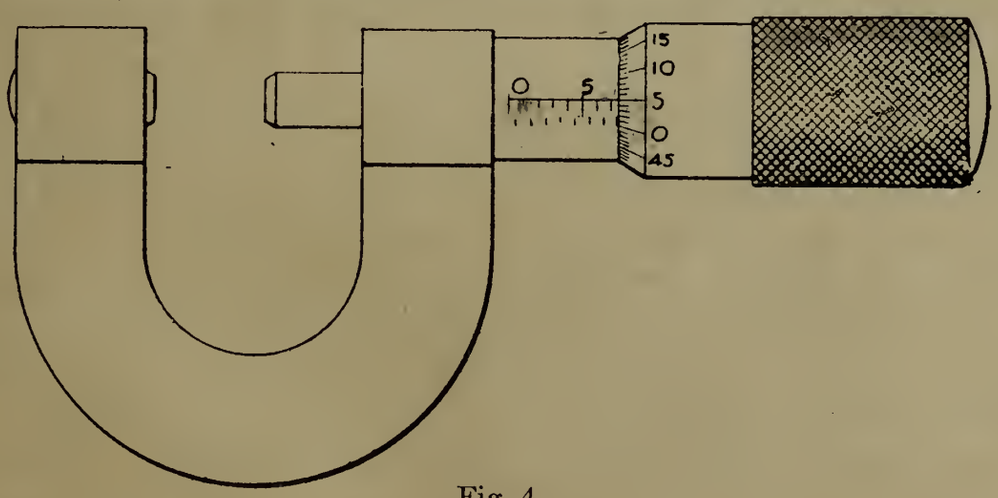


Fig. 4

frame, one end of which serves as a fixed jaw, and an accurate screw which serves as a movable jaw. The other end of the frame is threaded so that the screw can be turned in it,

toward or from the fixed jaw. It bears a straight scale whose divisions correspond to the pitch of the screw. The screw is turned by means of a milled head attached to a sleeve which moves over the straight scale and which has on its edge a circular scale whose equal divisions represent fractions of a turn. Usually the pitch of the screw is a half millimeter so that one turn of the screw advances it a half millimeter, the edge of the sleeve moving over one division of the main scale. In this case there are 50 divisions on the circular scale, each division representing one fiftieth of a half millimeter of 0.01 mm. In some cases the main scale is divided into millimeters, instead of half millimeters, each division corresponding to two complete turns of the screw. The object to be measured is placed between the fixed jaw and the tip of the screw. When these two are in contact the zeros of the two scales should be together. As the screw is turned the distance between them is indicated on the two scales, the number of millimeters on the straight scale, the fractional part of a millimeter on the circular scale.

In figure 4 the reading of the micrometer scales is 7.55 mm.

DIRECTIONS.—Place the slider of the balance at 0 on the scale and adjust the nuts on the horizontal screw so that the balance is in equilibrium, i. e. the pointer swings equidistant on each side of the center. Place the object to be weighed in one pan and add known weights to the other until the beam is very nearly balanced. Then the slider can be moved over the scale until equilibrium is again established. While adjusting the weights in either pan, the plunger under one of them should be held between thumb and fore finger in such a way as to form a cushion between pan and support, to prevent any jarring between them, otherwise the knife edges

may be thrown out of position and a readjustment for equilibrium will be necessary.

The slider reading is to be added to the known "weights" when they are in the right hand pan, but subtracted when they are in the left pan.

Weigh each of the three cylinders to 0.1 gram and record their masses in the proper place on the data sheet.

Measure the length of each cylinder with the vernier calipers, and its diameter with the micrometer calipers (if possible). The cylinders may not be perfectly circular and their ends may not be perpendicular to their axes so that measurements of the same dimensions taken at different parts of the same cylinder may differ slightly from each other. To get a good average measurement, the values recorded on the data sheet should be the mean of about 10 measurements taken at different parts of the cylinder.

To measure the length of the cylinder, first note the main scale divisions and determine the vernier fraction. If the zeros of both scales do not coincide when the jaws are together, note the zero error (the distance between them) and determine whether it is to be added to or subtracted from the readings to be taken. The cylinders should be placed with one end flat against the inner face of the fixed jaw, and its side in contact with the main scale, and the movable jaw should then be moved up flat against the other end, just touching it, and clamped in position while the reading on the scales is taken.

To take a measurement with the micrometer calipers, the scale divisions are first noted. Then the jaws are brought together, the milled head being held between the forefinger with slight pressure and turned until it just slips between the fingers instead of rotating. After noting the zero error as was done with the vernier calipers, the cylinder

is placed between the jaws and in contact with them, the screw being turned as before with slight pressure until it just slips through the fingers.

Note:—Never hold the milled head tight while turning the screw into contact as this will jam the threads and injure them, besides causing errors in the determination.

Care should be taken to measure the diameter and not a shorter chord of the cross section of the cylinder. Where the main scale is divided into millimeters, care must be observed to read fractional part of a division correctly, for evidently a reading on the circular scale may be so many hundredths of a millimeter or, 1/2 millimeter plus so many hundredths. In this case observe whether the edge of the circular scale is more or less than half a division from the last preceding millimeter mark. If this can not be determined by eye, remove the cylinder from between the jaws and turn the screw back until the circular scale reads zero. Then observe whether the edge stands at the millimeter mark or half a millimeter from it.

From these measurements the volume and the density of each cylinder are to be calculated and recorded on the data sheet.

The error in measurements due to the graduation of scales on instruments should be estimated in each case and recorded on the data sheet, and the maximum possible per-cent error in volume and in density, due to these errors in measurements, should be calculated.

QUESTIONS:—1—Determine from an examination of your calculations, which measurement causes the largest error in the density of each cylinder, and explain why.

2—A circular scale has divisions equal to 1/6 of a degree.

A vernier attached to it has a total length of 59 main scale divisions, and is itself divided into 30 parts. Show what the vernier fraction is in this case.

2. TESTING A SPIRIT LEVEL.

In this experiment we are to determine the radius of curvature of a level tube and to find the angle of tilt necessary to move the bubble through one division of the scale on the tube. (D. 183. G. 109. K. 175. W. 144.)

Apparatus.—Level tube, level tester and meter stick.

Description and Theory.—A spirit level is an instrument for detecting the deviation of a plane from the horizontal, and for measuring small angles. It is a glass tube having its inner surface accurately ground in an arc of a circle with a long radius of curvature. It is filled with a mixture of ether and alcohol except for a small bubble of air which always tends to move toward the highest point of the tube. A scale on the upper surface of the tube has divisions equal to 1/10 inch. When the tube is in a horizontal position the bubble is in the center of the tube, but when inclined the bubble moves through a distance which depends upon the angle of inclination and the radius of curvature.

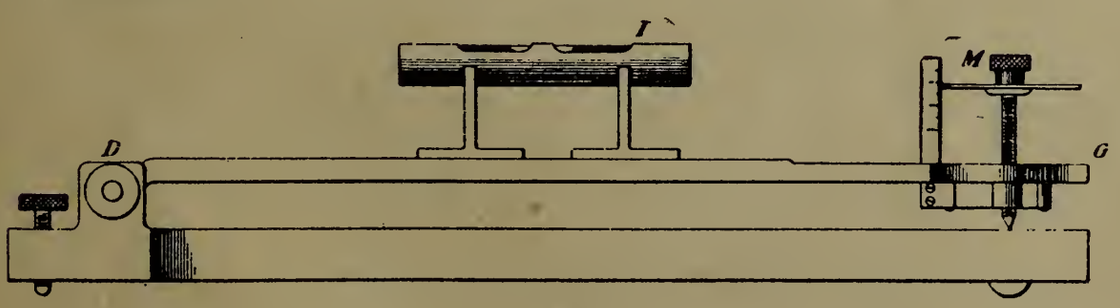


Fig. 5

The level tester consists of an iron arm *DG* hinged at one end into a heavy base and carrying a micrometer screw *M* at the other end. The glass level tube *T* is mounted in V grooves on the iron arm and may be tilted by turning the

micrometer screw. (See Fig. 5.) Attached to the level tester and parallel to the axis of the screw is a millimeter scale. A circular disc having its upper edge divided into 50 equal divisions is attached to the screw and moves along the straight scale as the screw is turned. It serves to indicate fractions of a turn. The pitch of the screw is a half millimeter, so that each of these divisions represents 0.01 mm.

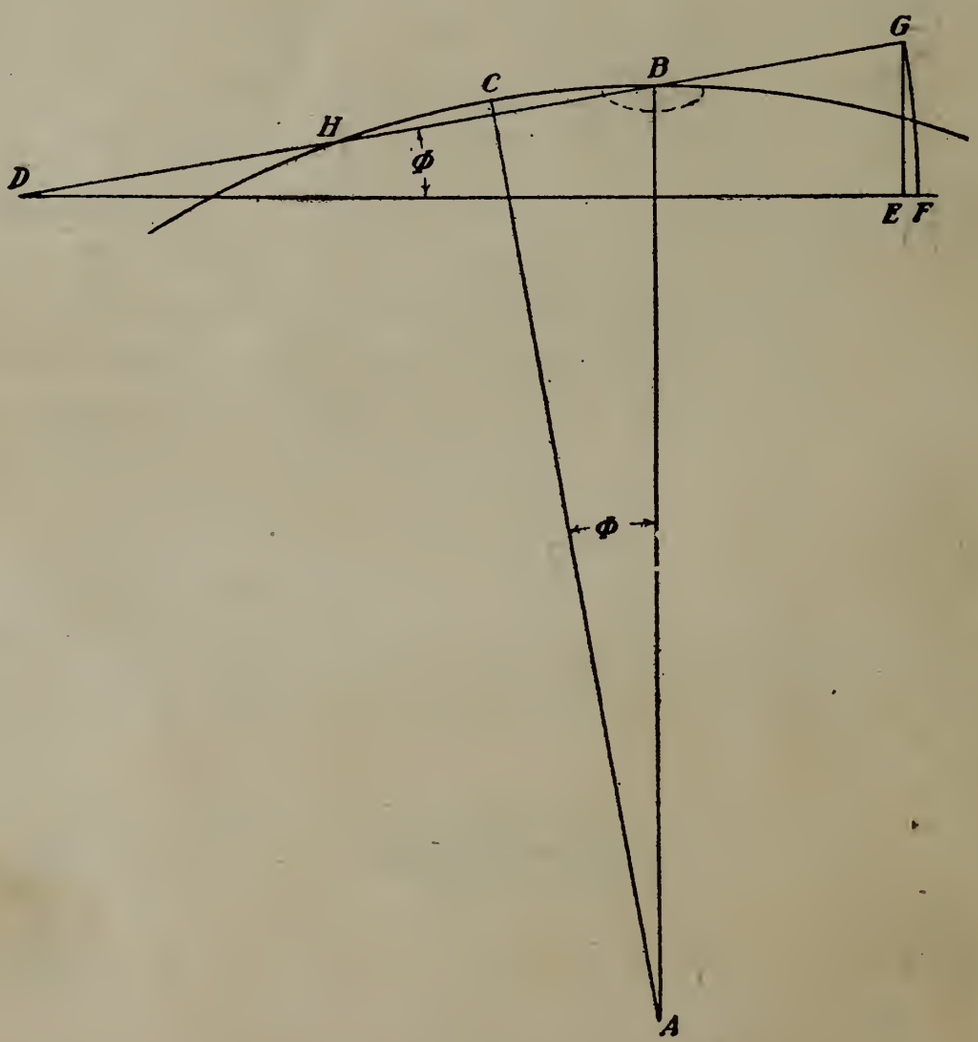


Fig. 6

In figure 6, let *BH* represent the inner curved surface of the tube and *AC* the vertical radius of curvature through the highest point (center of the bubble) when it is in the horizontal position, A being the center of curvature. If the

tube is inclined at an angle ϕ with the horizontal FD, the bubble moves from C to B, and the vertical radius through its center will be AB. The radius AC makes an angle ϕ with its former position. If the angle ϕ be measured in radians we have

$$\phi = BC \div AB \quad (1)$$

We have also the relation $\phi = FG \div GD$, or if ϕ is so small that the arc FG and the distance EG, through which the micrometer screw raises the end of the level tester, may be considered practically equal,

$$\phi = EG \div GD \quad (2)$$

It follows from (1) and (2) that

$$AB = BC/\phi = BC \times GD/EG \quad (3)$$

or the radius of curvature equals the distance the bubble moves multiplied by the ratio of the length of the iron arm to the vertical height through which its end is raised. Since the bubble moves through BC divisions when the tube is tilted through an angle ϕ radians, the angle necessary to move it one division will be $\theta = \phi \div BC$, but by equation (1)

$$\theta = \phi/BC = 1/AB \quad (4)$$

when AB is in scale divisions. Therefore the reciprocal of the radius of curvature gives the angle of tilt necessary to move the bubble through one scale division.

It should be noted that equation (4) can be true only when AB is expressed in scale divisions, hence the radius of curvature should be expressed in tenths of an inch for this calculation.

Directions.—Measure carefully the length of the iron arm from the center of the micrometer screw to the center of the hinge with a meter stick. As the ends of a straight scale become worn, the end divisions are usually inaccurate and measurements should be taken from some inside division near the end. The length should be estimated to 1/10 mm.

Next place the level tube in the V grooves and adjust the micrometer screw until one end of the bubble is just under a division mark at the end of the scale. Read the position of both ends of the bubble and of the micrometer and record them on the data sheet. Then turn the micrometer until the end of the bubble is at some division near the opposite end of the scale, and take readings as before. Repeat this operation five or six times, moving the bubble from one end of the scale to the other, and taking readings in both positions, but be careful to move the end of the bubble over the same number of divisions each time (or between the same two divisions). Calculate the mean displacement *BC* of the center of the bubble, and the corresponding mean height *EG*.

By means of equation (3) calculate the radius of curvature and express it in meters and in feet. By equation (4) calculate the angle of tilt for a movement of one scale division and express it in radians and in degrees. (One radian $= 57° 17' 45'' = 206265''$)

NOTE:—Be careful not to lean on the table, bench, or apparatus, nor to breathe on the level tube, nor to handle it more than is absolutely necessary and thus avoid errors due to a change in the size or position of the bubble. In order to diminish the error due to lost motion, the micrometer should be turned so as to raise the iron strip in each final adjustment before taking a reading.

QUESTIONS. 1—Which is the more sensitive, a level with a short radius of curvature or one with a long radius, and why?

2.—A carpenter's level with radius of curvature of 30 feet is placed on a board 10 feet long and the bubble position

noted. The spirit level is then reversed and the bubble moves 1.5 cm from its former position. How much in cm. must the end of the board be raised to level it?

3. EQUILIBRIUM OF COPLANAR CONCURRENT FORCES. COMPOSITION AND RESOLUTION OF FORCES.

(*a*) To show that when three concurrent forces are in equilibrium, each is equal in magnitude and opposite in direction to the resultant of the other two.

(*b*) To show that any one of these forces may be replaced by a pair of rectangular components.

(D. 95, 96, 102-105. G. 33-38. K. 46-49. W. 65-67, 71, 72.)

Apparatus.—Course balance and set of weights, 3 universal pulleys, 2 small buckets, can of shot, 4 iron weights, 4 strings with hooks attached, small ring, sheet of paper, 4 thumb tacks, dividers, and block of wood, vertical board and meter stick.

Theory and Method.—(*a*) Three weights are allowed to act simultaneously at a point. The resultant of two of these forces is found by parallelogram construction. The parallelogram law states that *when two concurrent forces are represented in direction and magnitude by the direction and length of two straight lines drawn from one and the same point, their resultant is similarly represented by that diagonal of the parallelogram constructed on these two lines as sides, which starts from the same point.* The resultant so found is compared in direction and amount with the third force.

(*b*) One of the forces is then replaced by a vertical and a horizontal force, which act at the same point as the original forces. These two new forces are adjusted in magnitude un-

til equilibrium is again established without altering the two remaining original forces. They are then compared with the vertical and horizontal components of the third force, as found by the parallelogram construction.

We now have four concurrent forces in equilibrium, but it should be noticed that the resultant of the two rectangular components (the original force) is equal and opposite to the resultant of the other two forces.

DIRECTIONS.—Find the mass of the weights on the coarse balance. Clamp a pulley on each side of the vertical board and fasten a sheet of paper to the latter with thumb

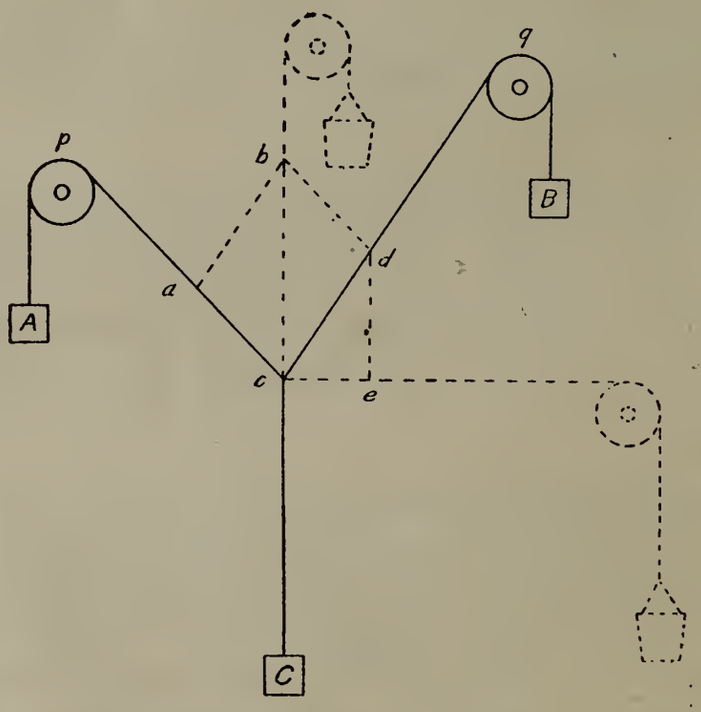


Fig. 7

tacks. Suspend the smallest and one of the larger weights (*A* & *B*) over the pulleys by means of the strings, and hook the other ends of the latter to the small ring. Suspend a third weight from this ring so that it hangs freely. The pulleys should be adjusted so that they turn in a plane parallel to the board and so that the weights do not touch the board or bench. Adjust the height of the pulleys so

that the center of the forces is in a convenient position for constructing the parallelogram of forces on the paper. Displace the weights slightly and allow them to move freely into a position of equilibrium. Then draw lines representing the forces under each of the strings, taking care not to alter the position of the latter by pressing against them. This may be done by carefully sliding a small rectangular block of wood along the paper until one *face* is in contact with the string near one end, without pressing against it, and drawing a short mark along this face with a sharp pencil. This is repeated at the other end of the string and a line is drawn through these two marks. The three lines drawn in this way should intersect at a point. Beginning at their intersection *c*, lay off on these lines to some convenient scale (say 1 mm. for each 5 grams) the corresponding forces. On the upper two lengths, *ca* and *cd*, as sides, construct a parallelogram *abcd*. The resultant of these two forces is the diagonal *cb*. Measure its length, calculate the value of the resultant in grams weight and compare it with the third force. Note also the angle between *bc* and the vertical. (*cC* prolonged upwards).

In the same way find the resultant of *A* and *C* and compare it with B.

It should be noticed in this construction that the sides of the triangle *bcd* are parallel to and proportional to the three forces. *Whenever three concurrent forces are in equilibrium, they are proportional to the sides of a triangle, the sides taken in order having directions parallel to the forces.*

(*b*) Lower the pulley *q* until its upper rim is in the horizontal line *ce*, and replace the weight *B* by a bucket partly filled with shot. Clamp a third pulley to the top of the board, adjust so that its rim is vertically above *c* and suspend a bucket and shot over it by means of a cord hooked to the

ring. Adjust the shot in these two buckets until the ring returns to its former position *c*, the strings *bc* and *ce* being respectively vertical and horizontal. Next weigh each bucket with its shot. Draw the horizontal line *ce* and drop the vertical *de* from the end of the line representing the force *B*. *ce* and *ed* now represent respectively the horizontal and vertical components of *B*. Measure them, calculate the value of the components in grams weight and compare these with the corresponding weights of bucket and shot.

NOTE:—For accurate results the parallelograms must be carefully drawn with a sharp pencil, and as large a scale as possible should be used in laying off the forces.

QUESTION.—If you had more than four concurrent forces in a system, how would you show that they were in equilibrium?

4. EQUILIBRIUM OF COPLANAR CONCURRENT FORCES BY THE TRIANGLE OF FORCES.

To verify the laws for three forces acting upon a point. (D. 52; G. 38; K. 48, 49; W. 72.)

APPARATUS.—Force table and accessories, known masses.

DESCRIPTION AND THEORY.—The force table consists of a cast iron disk whose edge is divided into degrees and which is mounted upon a heavy tripod (See Fig. 8). Three pulleys, working very free of friction in cone bearings, can be clamped to any part of the rim of the table. Known masses are fastened to light, flexible cords which pass over the pulleys and are attached to a small ring in the center of the table. While the masses are being adjusted the ring is held in place by a pin. If suitable masses *M*, *N* and *O* are placed on the holders at the ends of the strings, the pulleys

can be arranged so that the center of the ring remains over the center of the table when the peg is removed.

If a given point, say the center of the above ring, is held in equilibrium by three forces, the condition that must be

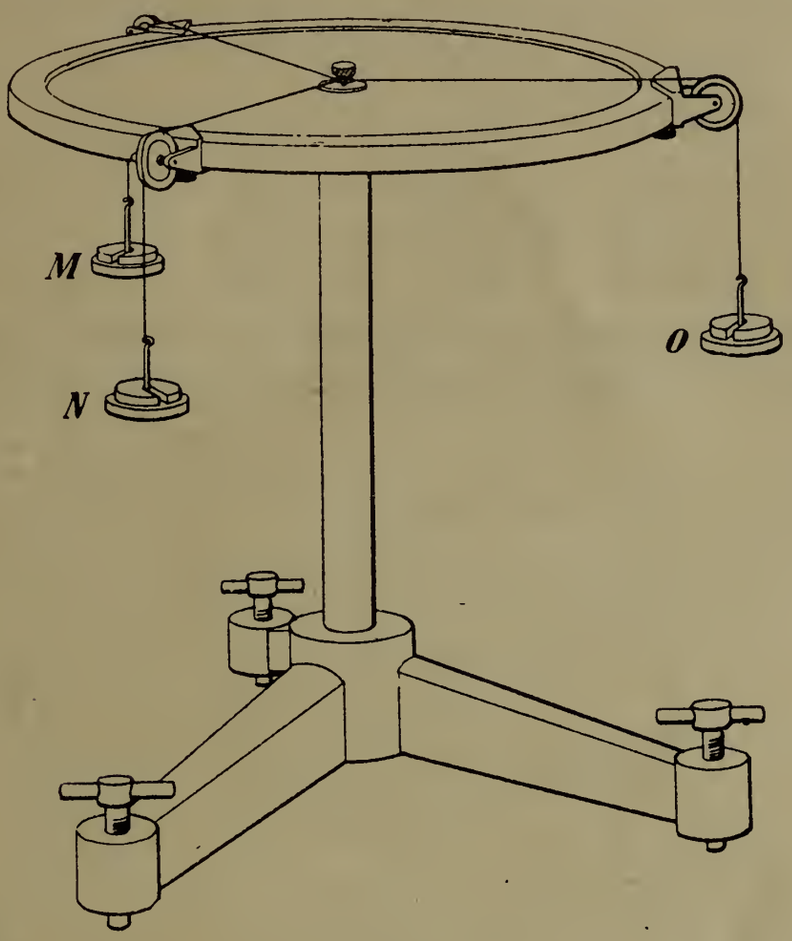


Fig. 8

satisfied is, that the forces are proportional to the sides of a triangle, the sides of the triangle being taken in the order of the forces and having directions parallel to the forces; or, in other words, the sum of the components of the forces resolved in any given direction must be zero.

Suppose for equilibrium, the forces take up positions as shown in Fig. 9 *a*. Let the forces be $X = Mg$, $Y = Ng$ and $Z = Og$, and the angles opposite the forces be $\propto$, β, γ, respectively. The condition for equilibrium states that the forces must form a triangle when lines proportional

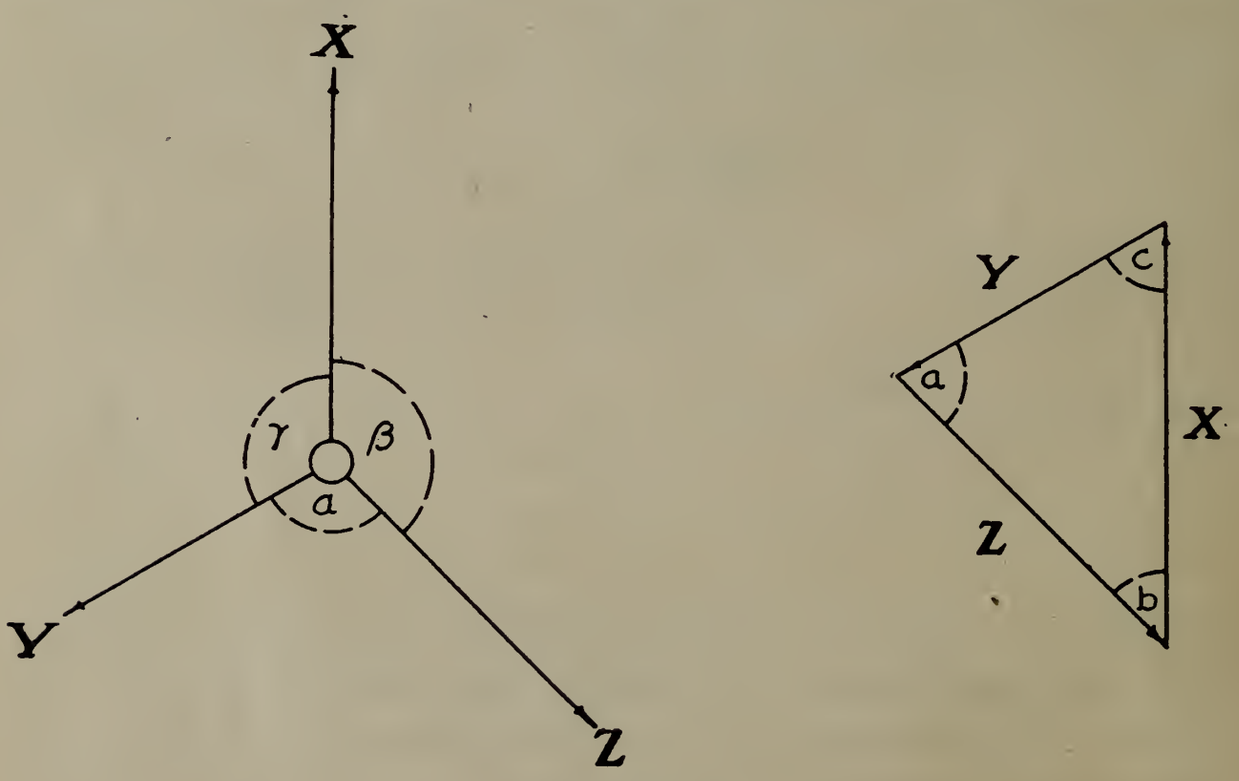
to the forces are drawn parallel to the directions in which the forces are acting. If the position of the forces be represented by Fig. 9 *a*, the state of equilibrium is represented by Fig. 9 *b*. From Fig. 9 *b*, we obtain, by trigonometry, the relation,

$$\frac{X}{\sin a} = \frac{Y}{\sin b} = \frac{Z}{\sin c} \qquad (1)$$

or

$$\frac{Mg}{\sin a} = \frac{Ng}{\sin b} = \frac{Og}{\sin c} \qquad (2)$$

Since the acceleration of gravity "*g*" enters each ratio, we can neglect it and use the values of the masses instead



Figs. 9a and 9b

of the forces, they being proportional to each other. The angles $\propto$, β, γ, are supplements of the angles *a*, *b*, *c*, respectively, and hence the final relation can be written

$$\frac{M}{\sin \propto} = \frac{N}{\sin \beta} = \frac{O}{\sin \gamma} \qquad (3)$$

Directions.—See that the force table is level. Place the small ring in the center of the table and put the pin through it to hold it in place. Fasten a flexible cord to each of the three weight holders, and, having run the strings over the pulleys, fasten them by means of the small hooks to the ring in the center of the table. Add masses of from 300 to 600 grams to each holder, but do not have the same mass on any two holders. Place one of the pulleys at zero and adjust the other two until approximate equilibrium is obtained, after which remove the pin from the center of the ring. Continue to adjust the pulleys until the ring remains over the center of the table when the latter is struck lightly with the hand.

When equilibrium is obtained and the center of the ring is over the center of the table, read the angles between the lines of action of the forces, or the angles formed by the strings with each other, and designate them as $\propto$, β and γ (Fig. 9 a). Determine the value of the masses at the end of each string (known masses plus the holder) and test the relationship expressed by equation (3).

Repeat the determination four more times using different combinations of the masses suspended each time. Find the mean value of the constant (mass divided by the sine of the angle opposite) for each of the five determinations. For example,—

First Determination

$$\frac{X}{\sin \propto} = \text{———}$$

$$\frac{Y}{\sin \beta} = \text{———}$$

$$\frac{Z}{\sin \gamma} = \text{———}$$

$$\text{Mean} = \text{———}$$

QUESTION.—(1) What is the maximum percent error from the mean, (a) in your best determination, (b) in your poorest?

5. EQUILIBRIUM OF COPLANAR NONCONCURRENT FORCES.

PARALLEL & RANDOM FORCES.

The object of this experiment is to become familiar with the laws of equilibrium

(*a*) for parallel forces acting in a plane.

(*b*) for random forces acting in a plane. (D. 81, 95-99, 102-105. G. 39, 40. K. 53-63. W. 68-73).

APPARATUS.—4 spring balances, 3 steel balls, 4 iron pegs, 2 boards, pair of dividers, 4 iron blocks, meter stick, 2 sheets of paper.

THEORY AND METHOD.—*In order that nonconcurrent forces may be in equilibrium they must tend to produce neither translation nor rotation.* Hence there are two rules for the equilibrium of such forces. We are going to consider only forces which act in the same plane. In this case—

I *If the forces are all parallel, the algebraic sum of the forces must equal zero, or the sum of the forces in one direction must equal the sum of the forces in the opposite direction.* If they are *not parallel* we may consider their components taken parallel to two rectangular axes, in which case *the sum of the components in one direction must equal the sum of the components in the opposite direction.*

II In either case *the algebraic sum of the moments of all the forces taken about any point in the plane must equal zero* in order that there may be no rotation. The *moment of a force* about a point is a measure of the turning tendency of

the force about an axis through that point. *It is numerically equal to the product of the force into the perpendicular distance from the point to the line of direction of the force.* A clockwise moment is one which tends to turn about the point in the same sense as the hands of a clock. An anti-clockwise moment tends to turn in the opposite direction. For equilibrium *the sum of the clockwise moments must equal the sum of the anti-clockwise moments.*

Both of these laws must be fulfilled by every system of coplanar forces when in equilibrium. We may find that a certain system does not produce rotation about a certain point. This does not necessarily mean that there is no rotation about some other point, or that there is no translation, hence we cannot say that the system is in equilibrium from this single test, but in general must apply both tests. Translation may be considered as rotation about an infinitely distant axis, so that the first law may be considered a special case of the second.

The apparatus consists of a base board with leveling screws, and a glass plate on its upper side, and an upper board having a series of equidistant holes in parallel rows. The upper board rests on three steel balls placed on the glass plate. The object of this arrangement is to avoid friction as far as possible. Small spring balances with their scales graduated in ounces are held by pins on the iron blocks and may be moved to any position relative to the board and to each other. By means of strings they may be fastened to pegs placed in any of the holes in the upper board and the tension on them may be adjusted by moving the iron blocks.

The spring balance consists of two cylinders one of which slides within the other, and which are held together by a spring. The extension of this spring is by Hooko's law pro-

portional to the weight suspended from it, and a pointer attached to the inner cylinder indicates this weight directly on the graduated scale of the outer cylinder. In a vertical position the inner cylinder is supported by the spring, whereas in the horizontal position it is not. Therefore the no load reading will be different in the two cases, and this difference must be added to the zero error if there is any, when the balance is used in the horizontal position.

DIRECTIONS.—(*a*)—Assume the balance marked A to be correct (without zero error) and find the zero error of the other balances by placing each on an iron block, joining it to *A* with a string, and pulling the two against each other in a straight line until a force of one or two pounds is indicated. Be careful in using the balances to have the inner cylinder in line with the outer cylinder to avoid errors due to friction between them. Note the zero error of the balance (the difference between the reading on it and that on the standard *A*), and mark it + or − according to whether it is to be added to or subtracted from future readings of the scale.

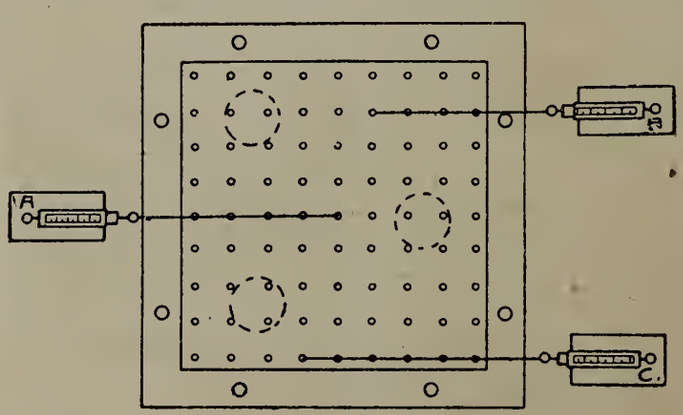


Fig. 10

Place the three balls between the glass plate and the upper board (as shown in figure 10) and adjust the leveling screws until the board stands at rest without touching the side stops. Arrange three balances as shown, fastening the strings to pegs which may be placed in any of the holes of

the upper board. Adjust the positions of pegs and balances until the pull on the single balance *A* is as large as possible and the strings and balances are in line and in the same horizontal plane, the strings being parallel to each other as shown by sighting along the parallel rows of holes. The upper board should rest on the three balls and be free from the side stops. Note the pulls on the balances, correct them for the zero errors, and record them. Next choose an arbitrary point on the board as a center and measure the perpendicular distance in inches from this point to the line of direction of each force, noting at the same time whether the force tends to rotate the board in a clockwise or an anticlockwise direction about the arbitrary center. Then calculate the moment of each force and record it.

On the data sheet compare the sum of the forces pulling in one direction, say east, with the sum of the forces pulling in the opposite direction, and the sum of the clockwise moments with that of the anticlockwise moments, giving the percent difference between them (calculated from their mean) in each case.

(*b*) Place a sheet of paper on the top board and arrange four balances so that the forces acting on the board are not parallel, but are placed at random, the iron pegs being punched through the paper. When equilibrium has been established (the balances being in line with their strings and the top board free from sidestops), rule a line under each string as in Exp. 3, and record the correct values of the forces pulling on the balances beside each line. Remove the pegs and paper and draw a pair of rectangular axes, say east and west, north and south. Lay off on each of the four lines to some convenient scale, the corresponding forces, and from the ends of the lines so laid off draw the components of the forces parallel to the two axes. Record

the values of the components and their direction. (*E. N.*, etc., for east, north, etc.)

Choose an arbitrary point *p* on the paper as a center and

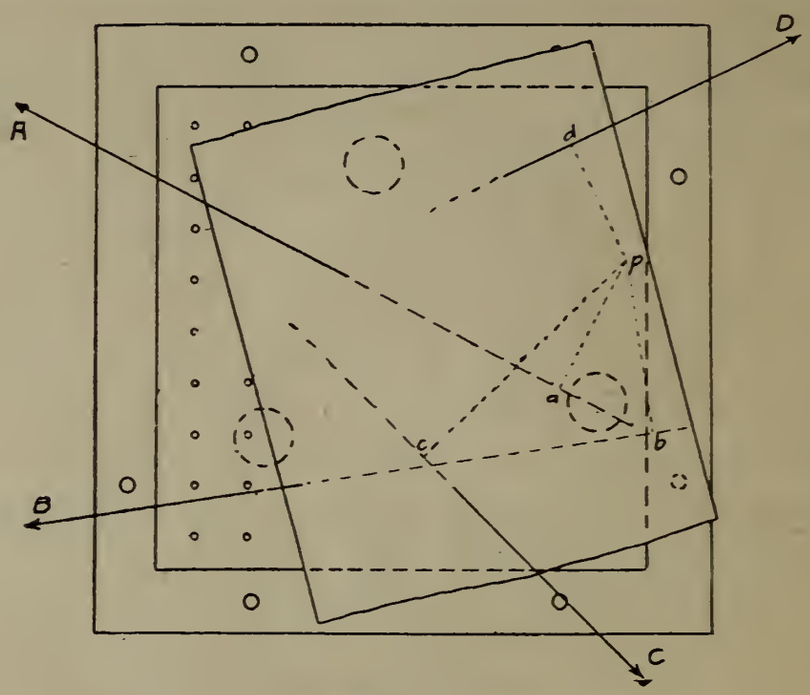


Fig. 11

measure the perpendicular distance in inches from this point to each of the force lines (extended if necessary as in Fig. 11). The arbitrary center should be so chosen that the perpendicular distances are as large as possible, to reduce errors in measurements, and the distances should be estimated to 1/64 of an inch. Note which forces give clockwise and which anticlockwise moments and record them properly.

On the data sheet compare the sum of the east components with the sum of the west components; the sum of the north with the sum of the south components; and the sum of the clockwise with the sum of the anticlockwise moments, and calculate the precent difference between them in each case.

QUESTIONS—1—A beam 10 feet long weighing 200 lbs., along which a pulley and chain travels, is supported at one

end by a wall, at the other by a column which cannot support more than 600 lbs. If the pulley with its load weighs 800 lbs. how close can it be drawn to the column without overloading the latter. Apply both laws of equilibrium.

2.—In performing this experiment with 4 random forces, it was found that two of them intersected in a point *O;* namely a force *A* of 10 oz. directed 30° W. of N., and *B* a force of 20 oz, directed 60° E. of N. A third force *C* of 30 oz. directed 45° E. of S. had a moment arm 3 inches long in a direction S. W. from *O*. Find the direction and magnitude of the fourth force and its distance and direction from *O*.

6. PARALLEL FORCES.

(*a*) To verify the law for the equilibrium of moments. (*b*) To determine the value of an unknown mass. (*c*) To find the mass of a meter stick. (D. 97-100; G. 39-41. K. 59-64; W. 69, 70.)

Apparatus.—Clamp stand, meter stick, set of known masses, three lever holders, coarse balance, unknown mass.

Description and Theory.—Moment of a force is the term used to express the value of a force times its arm, where the arm is the distance measured at a right angle from the line of action of the force to the axis about which rotation is capable of taking place. A body, free to rotate about an axis, is in equilibrium only when the two following conditions are satisfied: first, when the sum of the components of the forces in any direction is equal to the sum of the components in the opposite direction; and second, when the sum of the moments of the forces tending to produce rotation in one direction is equal to the sum of the moments of the forces tending to produce rotation in the opposite direction. The latter

condition, or law, may be verified by a simple arrangement consisting of a meter stick supported on a knife-edge at its center of gravity and having known masses supported on knife edges at such distances from the center of gravity that the product of one known mass plus the mass of the supporting knife edge times their distance from the center of gravity is equal to the corresponding product for the other mass.

The law can be applied to determine an unknown mass by replacing one of the known masses by the unknown. The product of the unknown mass times its distance from the center of gravity of the supporting meter stick is equal to the product of the known mass times its distance from the center of gravity. In all cases, the masses of the supporting knife edges which is stamped on the holder must be taken into account.

The mass of a meter stick can be determined by first finding the center of gravity of the meter stick by balancing it alone and then, having placed the center of gravity a short distance from the supporting knife edge, putting a known mass at such a distance from the supporting knife edge as to obtain equilibrium. The product of the mass of the meter stick times the distance of the center of gravity of the stick from the supporting knife edge must equal the product of the known mass times its distance from the point of support.

DIRECTIONS.—To verify the law for the equilibrium of moments, support the meter stick by one of the lever holders and adjust the same till equilibrium is obtained. Put a lever holder on each end of the meter stick and hang masses *A* and *B* (Fig. 12), of 200 and 300 grams on the holders. Adjust the position of the holders (at least 20 cms. from the supporting knife edge) with the suspended masses till equilibrium is obtained. Note the distance from the supporting knife edge to the knife edge of each of the holders.

Test the relation for the equilibrium of moments by multiplying the mass of the holder and the suspended weight by its distance from the supporting knife edge. The product of one suspended mass plus the mass of the holder

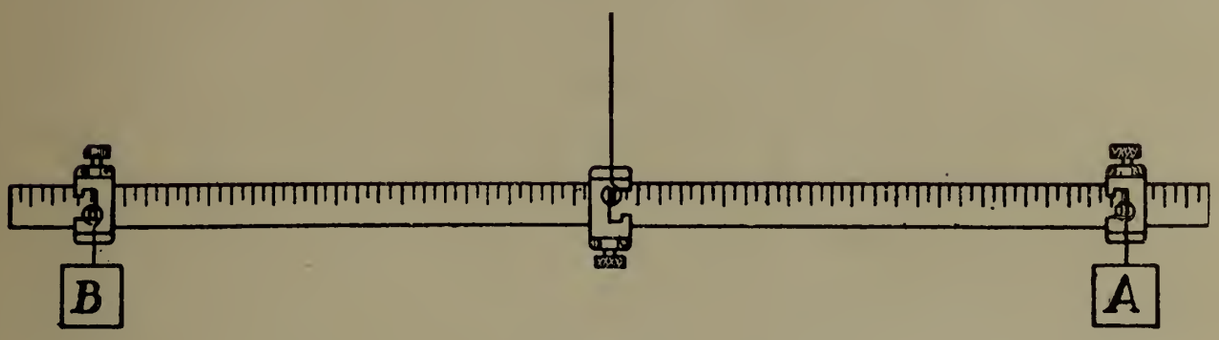


Fig. 12

supporting it times their distance from the supporting knife edge should be equal to the corresponding product for the other mass. Make a second adjustment of the masses A and B for different distances from the supporting knife edge and test the relation for the equilibrium of moments. Use another combination of masses 300 and 400 grams, and obtain the products of mass times the distance from the point of support for two different positions of the masses when in equilibrium.

In order to determine the value of an unknown mass it is only necessary to substitute the unknown mass X for either of the masses A or B above, and find the position for equilibrium. The unknown mass X is determined from the fact that the product of X times its distance from the point of support must be equal to the product of the known mass times its distance from the supporting knife edge. Make three different determinations of the value of X and find their mean. Remember that the masses of the knife edges supporting the known and unknown masses must always be considered.

The mass of the meter stick is found as follows: Find the position of the center of gravity of the meter stick by suspending it from one of the knife edges. Move the stick so that its center of gravity is from 15 to 20 centimeters from the supporting knife edge and suspend a 100 gram mass by means of a lever holder at such a position as to produce equilibrium. Note the distances of the 100 gr. mass and the center of gravity from the supporting knife edge. Then, if M is the mass of the meter stick, M times the distance of the center of gravity of the stick from the point of suspension is equal to 100 grams plus the mass of the holder supporting it times its distance from the knife edge of the holder supporting the system.

Make two more adjustments for equilibrium using different distances and determine the mass of the meter stick in each case. Calculate the mean of the three determinations of the mass of the meter stick.

QUESTION.—What two conditions must be satisfied for equilibrium of non-concurrent parallel forces? Show how, if the sum of all the forces downward did not equal the sum of the forces upward, the other condition could be fulfilled and still not have the bar in equilibrium. What kind of motion would we then obtain?

7. THE FINE BALANCE.

In this experiment we are to become familiar with the vibration method of using the fine balance. (D. 135; G. 73-76: K. 69, 70. W. 95.)

APPARATUS.—Set of weights (2mg. to 100 g.,) lead plate to be weighed, spirit level, and fine balance.

DESCRIPTION AND METHOD.—The fine balance is to be

used whenever an object is to be weighed to less than 0.1 gram. It consists of a rigid beam with a central knife edge resting on a polished agate plate at the top of a pillar, and two equidistant knife edges *KK* from which the balance pans are suspended by means of stirrups with polished agate plates. (See Fig. 13.) Agate is used in order to make the friction in swinging very slight. To protect the balance from disturbing air currents and dust, it is enclosed in a glass case having a heavy glass base and supported on leveling screws. To avoid jarring and chipping of the knife edges when the balance is not in use or while "weights" or objects are being added to or taken from the pan, an arrestment

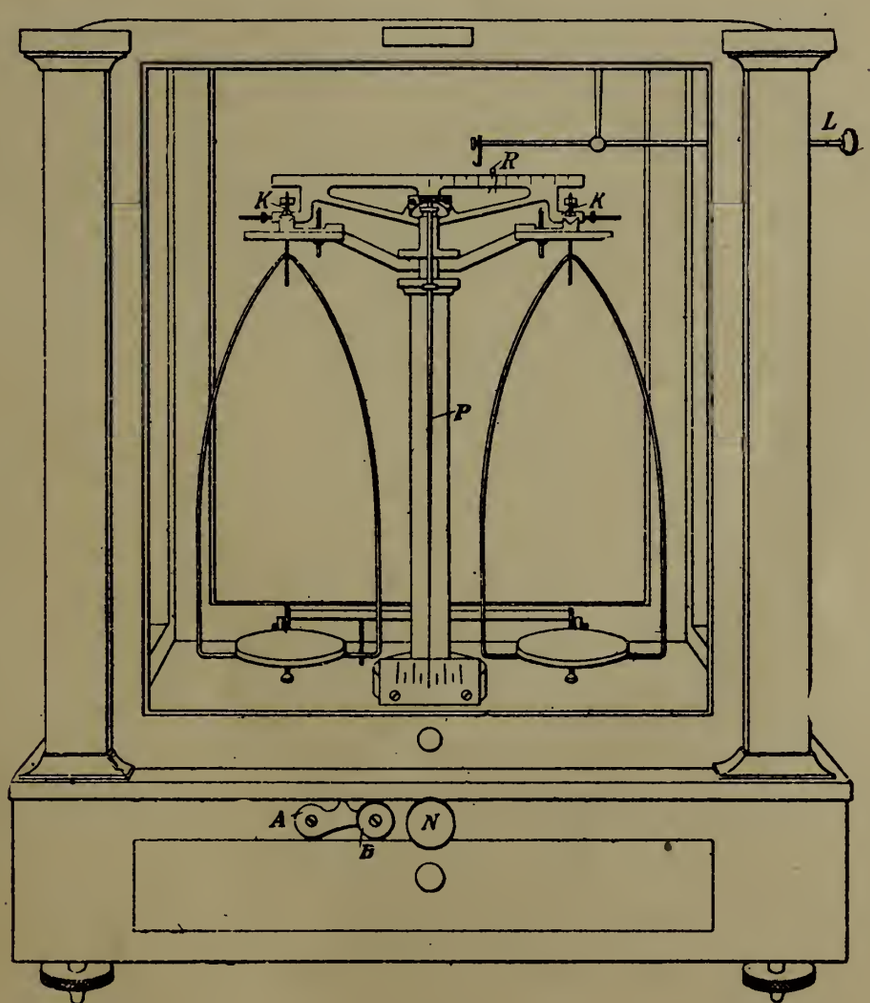


Fig. 3

device is used. This is operated by a thumb nut (*N*) at base of balance which, on turning, lifts the beam off the agate plate and the stirrups off the knife edges. Two stops rest against the pan to prevent them from swinging. To release them a button (*B*) is pushed in until it is caught by the lever (*A*). A long pointer (*P*) attached to the beam indicates the position of rest on a scale at the base of the pillar.

As the milligram weights are quite small and inconvenient to handle, a rider (*R*) is often used in place of weights less than 10 mg. It is a piece of wire which weighs exactly 6 mg. and is so shaped that it may be placed astride of a scale on the beam. By means of a system of levers operated from the outside by a thumb nut (*L*) at the right of the case, this rider may be moved to any point on the scale. When just above one of the end knife edges its effect is the same as if it were placed in the pan. When placed at one sixth this distance from the central knife edge its moment about the fulcrum will equal that of one sixth of its mass, or (1 mg.), placed in the pan. The marked divisions on the scale represent milligrams, the smallest divisions tenths of a milligram.

Adjustable nuts working on horizontal screws at the ends of the beam, make it possible to balance the latter alone.

A good balance should fulfill the following conditions: (1) The balance must be true, that is, the beam must remain horizontal whenever equal masses are placed in the scale pans. (2) It must be sensitive, i. e. small differences in the masses in the two pans should cause an appreciable deviation of the beam from its horizontal position. (3) It must be stable; that is, after the beam has been disturbed from its equilibrium position, it must return to it. (4) It

should have a short period; i. e. when the beam has been disturbed and oscillates before coming to rest, the time required for it to make a complete swing should be short.

In order that the balance may be true, the beam should be inflexible and its arms of equal length. To be sensitive the knife edges should be parallel and in the same plane, the arms should be long, and the center of gravity of the beams should be just below the central knife edge. In order that the beam may quickly return to its equilibrium position when displaced, the arms should be short and the distance between center of gravity of beam and knife edge large. These conditions conflict with those for sensitiveness and in practical balances a compromise between them must be chosen.

The approximate weight of an object may be obtained by the equal swing method. Small bits of paper are added to the pans until the pointer swings equidistant on either side of the center division. Then the object is placed in the center of one pan and weights are added to the other until the pointer swings as nearly as possible as before, equidistant on either side of the center division. Then the object and known weights will have approximately the same mass. To find the mass more accurately, it would be necessary to find the true resting point (*i. e.* the point on the scale at which the pointer would finally come to rest if allowed to swing freely). As the period of the balance (the time required for a complete vibration of the beam) is usually long, it is preferable to use the following "vibration method of weighing". The resting point with no load in the pans is found by observing an odd number, say five, consecutive turning points of the pointer, estimating to tenths of a scale division. The reason for taking an odd number of turning points is the following:—The amplitude of swings to either side of

the resting point is continually decreasing by approximately equal amounts, say a divisions for a few equal swings (see Fig. 14). The mean of the first and second turning points would be approximately $a/2$ divisions to one side of the resting point. The mean of the second and third turning point would be about $a/2$ divisions to the other side of the resting point. The mean of two means would be very near the resting point. If five turning points were observed instead of three, the mean of the means of the five readings, taken two and two consecutively, would be still nearer the point, whereas if any even number of points or an odd number of swings were observed the mean would always be to one side or the other of the true resting point. Therefore, to find the true resting point, average the mean of n (odd) turning

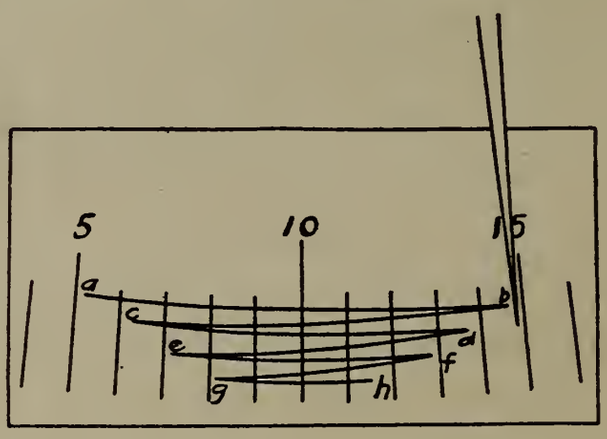


Fig. 14

points on one side (a, c, e, Fig. 14.) with the mean of $(n-1)$ intervening turning points on the other (b, d, Fig. 14). As an example, if the pointer swings from 6.0 to 12.8 and back to 6.8, the two means would be 9.4 and 9.8 respectively and the mean of the two means of resting point would be 9.6. This no load resting point we will call S_o. If now the object to be weighed is placed in one pan and known weights in the other until they balance, the load resting point S_1 may be found in the same way as S_o and in general will

be different from S_o. But it is evident that the masses in the pans can only be equal when the pointer comes to rest at S_o, since this is its equilibrium position. We must therefore determine what weight must be added to or taken from the known weights in the pan in order to make the pointer come to rest at S_o (instead of S_1) when the balance is loaded. To do this we add a small weight, say 5 mg. to the known weights and observe the new load resting point S_2. Since 0.005 grams moves the resting point from S_1 to S_2 the number of grams n necessary to move one division will be $0.005 \div (S_2 - S_1)$. The reciprocal of n is called the sensibility of the balance; *i. e.* the number of divisions through which the pointer is moved by 1 mg. in the pan. In order that the load resting point may be changed from S_1 to S_o we must change the known weights by $n\ (S_1 - S_o)\ g$. If the known weights are in the right pan we must add this amount to them when S_1 is greater than S_o and subtract it when S_1 is less than S_o, in order to obtain the true mass of the object.

Directions.—*Caution.* Always keep the case closed except when it is necessary to adjust the weights. Lower the arrestment slowly and carefully, avoiding so far as possible, all jarring of the balance. Always raise the arrestment before changing the weights in the pans. Handle the weights with tweezers to avoid corroding them or changing their weight. Do not attempt to make any of the finer adjustments with the screws on the balance. If such adjustments are necessary call on one of the instructors to make them.

Adjust the leveling screws until the base of the balance is horizontal as shown by the spirit level. Lower the arrestment until the pointer swings freely and note whether the no load resting point is within one or two divisions of the center. If it is not, drop small bits of paper into the proper pan until it is within this distance from the center. Then

find the no load point S_o accurately, by taking five consecutive turning points, (estimating to 1/10 of a scale division) and averaging the mean of those on one side with the mean of those on the other side. Next raise the arrestment and place the object to be weighed in one pan and known weights in the other to balance. Lower the arrestment a trifle and note which way the beam tilts (*i. e.* which of the pans has the heavier mass in it); then raise the arrestment and adjust the weights to secure a better equilibrium. Repeat until there is very little tendency to tilt the beam. Time may be saved by trying the weights in the order in which they come in the box, and replacing them in similar order. The finer adjustments may be made with the rider. Record the mass m of known weights.

Determine the load resting point S_1 from five consecutive turning points. Add 5mg or less to the known weights, determine the load resting point S_2, and calculate n, the reciprocal of the sensibility of the balance. Remove the weights and object from the pans (while the arrestment is raised, of course) and determine the no load resting point S_3. If the latter differs by less than one division from first value S_o the weighing is sufficiently accurate. If the difference is greater than one division the experiment should be repeated.

From the data taken, calculate the true mass M of the object. $M = m \pm n\,(S_1 - S_o)$

QUESTION.—If the center of gravity of the beam could be raised or lowered by means of an adjustable nut, how would the balance be affected in each case.

8. THE PENDULUM.

(*a*) The laws of the simple pendulum.

(*b*) The determination of the acceleration of gravity

with the Kater's pendulum. (D. 117, 120, G. 56, 81-84, K. 127, 128, 141, W. 112-118.)

Apparatus.—Two lead balls, fine thread, vernier calipers, reversible pendulum and support; beam compass, meter stick, and seconds clock.

Description, Method and Theory.—(*a*) If it were possible to have a material particle concentrated in a point and suspended by a weightless, inextensible thread, this would be a theoretical *simple pendulum*. These conditions can not be fulfilled in practice, and the practical simple pendulum consists of a small heavy bob (usually spherical) suspended by a light and practically inextensible thread or wire. Fig. 15. The period of such a pendulum is the time required for a complete oscillation, or double swing; that is, the time which elapses between two successive passages of the pendulum *in the same direction*, through some point of its path, such as the center of its swing. The length of "the equivalent simple pendulum" (that is, the length of the theoretical simple pendulum which has the same period) is, for a spherical bob (neglecting the mass of suspension),

$$l = d + (2r^2/5d)$$

where d is the distance from the point of support to the center of the bob, and r is the radius of the bob. For practical purposes the length of the pendulum is taken as equal to d. The period of such pendulum is

$$T = 2\pi\sqrt{l/g},$$

where g is the acceleration of gravity. The period is independent of the mass of the bob and of the arc of its swing, provided that arc does not exceed about 4 degrees.

We wish to test the variation of the period of such a pendulum with its length and with its mass. In order to find

the variation of any physical quantity with one of its factors, it is evident that all other factors, must be kept constant. We therefore take two pendulums of equal mass but of different lengths, in order to compare the variation of their periods with their lengths, and two pendulums of equal lengths but of different masses, to note a possible variation of period with mass. If the student is working alone he may determine the period of the pendulum with a stop watch,

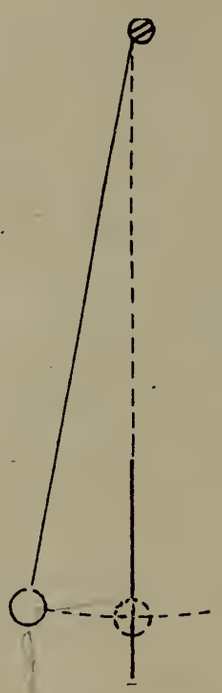


Fig. 15

by starting and stopping the watch at the beginning and end, respectively, of a definite number of oscillations. If two students work together, an ordinary watch with a seconds hand may be used, one student observing the number of oscillations of the pendulum, the other the number of seconds which elapse. The first student should call "ready" a few seconds before he begins to count the oscillations. Then at the exact instant when the pendulum passes through its center of path (position of rest) he should call "start", beginning at the same time to count the passages of the pendulum through this point *in the same direction*, starting with

zero for its first passage. The other student should note the exact time at this instant. On the 95th or 96th passage the first student again calls "ready" to warn the second student, and on the instant that the pendulum completes its 100th passage through the position of rest he calls "stop", upon which the second student again notes the exact time. If the students have no watch at hand, they may observe the time on the seconds clock in the laboratory. This clock has a pendulum which completes one *swing* (half oscillation) every second. It has a device for closing an electric circuit every time it passes through its position of rest. In this circuit are included a galvanic cell and a telegraphic sounder, and every time the circuit is closed, the sounder gives a loud click, thus indicating seconds. When all is ready for the trial one of the students calls "ready". The second student then observes both the passage of the pendulum through its position of rest and the click of the sounder. At the instant when both events occur exactly in unison *he* calls "start", beginning at the same time to count the clicks of the sounder, zero, one, two, three, etc., while the first student counts the passages of the pendulum in the same direction, beginning zero, one, two, etc. When nearly one hunderd swings have been completed the first student begins to observe the clicks of the sounder as well as the passages of the pendulum and when he sees that the two nearly coincide he calls "ready", and at the exact instant when both occur simultaneously he calls "stop", noting the exact number of passages, while the second student notes the exact number of clicks up to this instant. To avoid the tedious counting of oscillations we may proceed as follows:—

Note as accurately as possible with a watch or with the seconds clock the time of 10 complete oscillations, and calcu-

late the approximate period. Then note the exact instant of the transit of the pendulum through its position of rest. Calculate the approximate time required for 100 oscillations, and at the expiration of this time again note the exact instant of transit in the same direction as before. Divide the interval of time between these two transits by the approximate period. The whole number coming nearest to this quotient will be the exact number of oscillations in this interval and on dividing the same interval by the exact number of oscillations we get the exact period.

The length of the pendulum is measured with a beam compass which consists of a straight wooden beam along which two trammel points may be moved and clamped at any desired position. The distance between the trammel points may be measured with a meter stick.

The average time of one complete oscillation should be calculated and a comparison made between the periods for different lengths and for different masses.

(*b*) The compound pendulum differs from the simple pendulum in that its mass is not concentrated at one point. Evidently every physical pendulum is really a compound pendulum. That point in the pendulum where the particles are oscillating with the same period that they would have if they were bobs of simple pendulums suspended from the same point of support as the actual pendulum, is called the center of oscillation, the point of support being called the center of suspension. These two points are interchangeable. That is, if the pendulum is suspended from the center of oscillation, the previous center of suspension becomes the center of oscillation. If the whole mass of the pendulum were concentrated at the center of oscillation, the period of the resulting simple pendulum would be the same as that of the actual pendulum. Therefore the length of "the equivalent sim-

ple pendulum'' is the distance between the center of oscillation and the center of suspension. The fact that these two centers are interchangeable makes it possible to determine their relative positions accurately and so makes possible the accurate determination of the length of the equivalent simple pendulum. The period may also be determined accurately

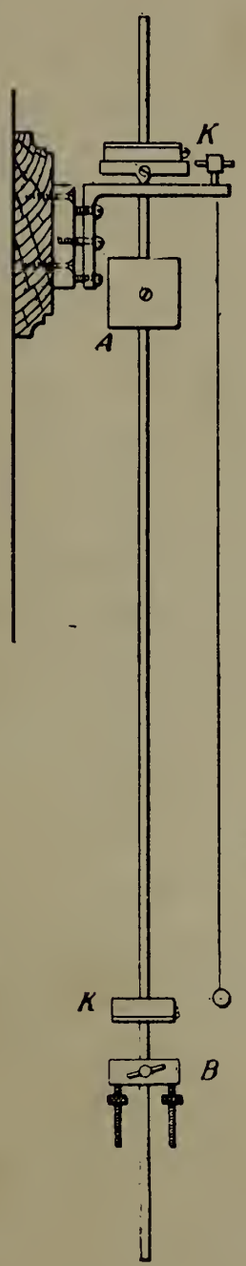


Fig. 16

and consequently an exact determination of the acceleration of gravity is possible from the relation

$$g = 4\pi^2 l / T^2$$

(see expression for period of simple pendulum).

For this purpose a Kater's reversible pendulum is used. It consists of a metal rod, carrying two adjustable steel knife edges *KK* with their edges turned toward each other. See Fig. 16. At one end a heavy mass *A* is fixed, while a light mass *B* with adjusting nuts may be clamped at any position along the length of the pendulum. The knife edges are fixed at some definite distance apart and the pendulum is swung first from one knife edge, then from the other, its period being determined each time and the weights adjusted until the periods are exactly equal. The period and the length between the knife edges may now be accurately measured and the acceleration of gravity calculated.

DIRECTIONS.—(*a*) *Relation between period T and length l.*—Fasten the smaller ball to the thread and suspend it from the support. Adjust the length of the thread until it is about 60 or 64 cm. Adjust the pointer on the support until it is just under the center of the bob when the latter hangs at rest. Place the beam compass in line with the pendulum, one end resting on the floor, and adjust the trammel points until they stand exactly at the ends of the thread. Lay the beam compass on the table and measure the distance between the trammel points by means of a meter stick and record it on the data sheet. Measure the diameter of the bob with the vernier calipers in four or five different positions, and record the mean diameter. Then calculate the length of the pendulum.

Set the pendulum to swinging through a *small* arc. Be careful to have it swinging in a plane; that is, so that the path of the bob is a straight line, not an ellipse. Find the time for one complete vibration by the method explained above, noting the passage of pendulum past the pointer. Repeat the trials until three values of the period are obtained

which do not differ by more than 0.01 second and then record the mean of these trials.

Repeat these measurements with a pendulum about one meter long.

Show by your data that the period is directly proportional to the square root of the length, by calculating the ratio between them in each case. Record these ratios and the percent difference between them calculated from their mean.

Independence of Period and Mass.—Fasten the large lead ball to the thread and suspend it as before. Keeping the position of the trammel points on the beam compass exactly as they were for the last measurement of length, adjust the length of the pendulum until the center of the bob is at the lower trammel point. The pendulum then has very nearly the same length as the previous pendulum with the smaller bob. Proceed to find the period of the pendulum and compare it with that of the previous pendulum of the same length. Record these periods and the percent difference between them calculated from their mean.

(*b*). *The determination of g.*—Using the Kater's pendulum let one knife edge and the two weights remain fixed, and the second knife edge K movable. Determine the period of the pendulum about both knife edges for one position of K, and measure the distance "l" between knife edges. Then shift K about 5 cm. and repeat the measurements. Repeat until you have four periods about each knife edge and the corresponding distances between them. Plot two curves having values of l as abscissae and corresponding periods about both knife edges as ordinates. These two curves will intersect and the ordinates of the point of intersection will be the period and length of the equivalent simple pendulum respectively.

For accurate results the distance between knife edges

should be measured with a comparator, but for our purposes we may use the length measured with the beam compass and meter stick.

From the period and length of the equivalent simple pendulum calculate the value of g and record it on the data sheet.

QUESTION.—In the case of the third simple pendulum used (with the larger bob) what is the percent error in the length due to the assumption that the length equals d?

9. LAW OF FALLING BODIES.

The object of this experiment is to observe the time and distance of the free fall of a ball and, applying the laws of uniformly accelerated motion in a straight line, to determine how accurate a value of the acceleration of gravity can be found from such observations. (D. 26-29, G. 51, K. 96-99, W. 35.)

APPARATUS.—Falling body apparatus, beam compass, meter stick, tray, steel ball, terminals of storage battery and regulating resistance.

THEORY, METHOD AND DESCRIPTION.—If a body falls freely a distance d in a time t, we have the relation

$$d = 1/2\ gt^2$$

where g is the acceleration due to gravity. With the apparatus we measure corresponding values of d and t and since g is constant we can test the above law by seeing whether the ratio of corresponding values of d and t^2 is constant. We can also substitute corresponding values in the formula and get a value for g.

In the experiment, a small ball B is held by an electromagnet A (See Fig. 17.) A pendulum KWP is hung from

the knife edges K just under the ball, so that, when the latter is released, it falls through a slit X in the platform P at the lower end of the pendulum when the pendulum is at

Fig. 17

rest. The pendulum can be held at one end of its swing by a second electromagnet C. Both A and C are in the same electric circuit, so that when the current is broken the ball and pendulum are released at the same time. The distance of fall can be varied by raising or lowering the rod which carries the magnet A, until it is at such a position

that the ball drops through the slit X as the pendulum swings through its position of rest. The time of fall then equals the time required for the pendulum to swing from one end of its path to its position of rest, which is one quarter of its period. The corresponding values of d and t are then to be measured. The period may be altered for a second trial by shifting the iron frame W to a different position.

DIRECTIONS.—The battery circuit is connected through a resistance and a snap switch to the binding posts on the fall apparatus. Adjust the current by increasing or decreasing the resistance until the electromagnets are just strong enough to hold the ball and the pendulum. Slip the frame W to the bottom of the pendulum and adjust the position of the slit X so that the ball drops through it when the pendulum is at rest and the width of the slit is not over one fourth of a millimeter greater than the diameter of the ball. Then by adjusting the height of the upper magnet, find the position for which the ball will slip through the slit of the *swinging* pendulum when ball and pendulum are released simultaneously by snapping the switch *quickly*. One jaw of the slit is of lead, the other of brass, so that one can tell by the click on which plate the ball strikes, hence whether it strikes in front of or behind the slit. In the first case the magnet should be raised; in the second it should be lowered. The tray should be placed under the apparatus to catch the ball as it drops. When the height of the ball has been properly adjusted, measure the distance d from the center of the ball to the slit with the beam compass and meter stick. Then set the pendulum in vibration and determine the time of 50 complete swings. Repeat this a second time and calculate the mean quarter period of the pendulum.

Change the position of the frame W so that the period and distance d are altered, and repeat the experiment.

With these values of d and t calculate the acceleration of gravity by the formula given.

Finally determine by about ten trials the upper and lower limits of the height through which the magnet A can be shifted and still allow the ball to pass through the slit in the swinging pendulum.

Question.—What is the maximum possible percent error in the calculated value of g due to this possible variation in the height d determined by the last measurement?

10. ACCELERATION OF GRAVITY WITH A FALLING TUNING FORK.

To determine the value of the acceleration of gravity. (D. 26-29; G. 81; K. 96-99; W. 35.)

Apparatus.—Acceleration apparatus, meter stick.

Description and Theory.—The acceleration apparatus (Fig. 18a) consists essentially of a tuning fork attached to a frame arranged to fall about one meter between two vertical guides which offer as little resistance as possible. The fork is held at the top of the guides by a trigger by means of which it can be released and at the same time set in vibration. On one prong of the tuning fork is a stylus for recording the vibrations upon a long plate of smoked glass in the nature of a curve such as is shown in Fig. 18b.

If the point a (Fig. 18b) near the beginning of the tracing, where the waves are perfectly formed, is selected and spaces of exactly six wave lengths marked off, it is seen that any set of six wave lengths is longer than the set of six just preceding it. The time required for the tuning fork to pass over the distance is the same, however, because the fork has made six complete vibrations in each case.

When the fork had reached the point a it had acquired a velocity which may be represented by v_o. If t represents the time for the fork to make six complete vibrations, it will

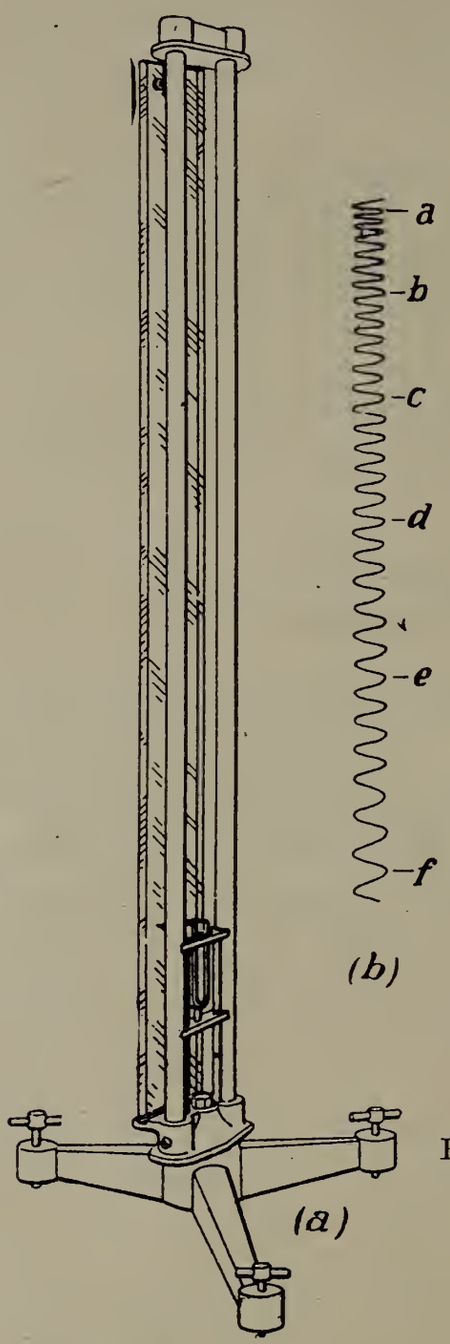


Fig. 18a and b

pass over a distance $v_o t$ during the interval t because of its initial velocity alone. But gravity also acts and will increase the space passed over in the interval t by the amount $\frac{1}{2}gt^2$. The total space passed over in the first interval a to b is then,

$$ab = v_o t + \tfrac{1}{2} g t^2 \qquad (1)$$

In two intervals of time t, (*i. e.* $2t$), the tuning fork will pass over a distance given by

$$ac = 2v_o t + \tfrac{1}{2} g (2t)^2 \qquad (2)$$

and for three intervals

$$ad = 3v_o t + \tfrac{1}{2} g (3t)^2 \qquad (3)$$

The differences ab, $ac - ab$ and $ad - ac$ are the distances passed over in successive equal intervals of time. The distances are given by the following equations:

$$ab = v_o t + \tfrac{1}{2} g t^2 \qquad (1)$$

$$ac - ab = bc = v_o t + \tfrac{3}{2} g t^2 \qquad (4)$$

$$ad - ac = cd = v_o t + \tfrac{5}{2} g t^2 \qquad (5)$$

From these equations it is seen that the *increase* in the space passed over in successive intervals is

$$bc - ab = cd - bc = g t^2 \qquad (6)$$

This states that the difference between the distances represented by any two successive groups of six wave lengths is given by the value of gravity times the square of the time consumed by the tuning fork in making six complete vibrations. The value of g, the acceleration of gravity, can be determined if the time t required for the fork to make six complete vibrations and the difference between the distances passed over in two successive six vibration intervals are known. The time t is obtained by dividing the number of vibrations made in the time t by the number made in one second which is recorded on the fork, or

$$t = \frac{6}{n} \qquad (7)$$

where n is the pitch of the fork. The difference between successive distances can be obtained by direct measurement.

Directions.—In order that the friction may be as little as possible the uprights should be standing vertically. Put

the smoked glass plate in position moving it towards one side so that the first tracing made will be near one edge. Fasten the tuning fork on the eccentric at the top of the frame and, having the stylus pressing closely against the glass plate, turn the lever connected to the eccentric so as to release the fork. The fork will fall freely and, in so doing, make a tracing upon the smoked glass. In a similar manner make at least six tracings on the plate, shifting the position of the plate each time so as to have each tracing separate from the others. Remove the plate from the frame and, beginning with the first good wave, mark off spaces of six wave lengths on each tracing. Measure with a millimeter scale all of the six vibration spaces on each tracing that it is possible to obtain, and find the difference between successive distances represented by six wave lengths. Find the mean value of the differences for each of the tracings.

Note the pitch of the fork as recorded upon it and, substituting the pitch n in equation (7), find the value of t. Substitute the mean value of the differences for one of the waves and the value of t in equation (6) and solve for g. In like manner determine the value of g as given by the data of each of the other curves. Calculate the mean of the values of g thus found.

QUESTIONS.—What effect will friction of the falling frame have on the value of g? Explain your answer. What percent error in g will be caused by an error of one percent in n, the frequency of the fork?

11. FRICTION.

To study the laws of sliding friction and to determine the coefficients of static and of kinetic friction. (D. 126-130, G. 49, K. 80-84, W. 96-99).

Apparatus.—Friction table, block of wood, strip of aluminum or glass, coarse balance and weights, small can and thread, metal shot and spirit level.

Method, Theory and Description.—By friction we mean the resistance to motion between two bodies in contact, caused by their surfaces. This resistance is independent of the area of contact between the two bodies but is directly proportional to the total pressure between them. In general, friction between two surfaces of the same substance is greater than friction between two surfaces of different substances. When the two bodies are at rest, the force necessary to make one slide horizontally upon the other must exceed a certain definite value, which is called the "maximum static friction". *The ratio of the maximum static friction F between two surfaces, to the normal pressure N between them is called the "coefficient of static friction".*

After the one body has started to slide over the other a certain force is necessary to keep it in uniform horizontal motion. *The ratio of this friction force to the total pressure is called the "coefficient of kinetic friction"* ϕ.

When one of the surfaces is a plane inclined to the horizontal and the angle of tilt is gradually increased, there is a critical angle "i" at which the second surface will slide uniformly down the plane when given a start. *It may be shown that the tangent of this critical angle is numerically equal to the coefficient of kinetic friction,* or

$$\tan i = \phi$$

The friction table Fig. 19 consists of a heavy base board mounted on leveling screws, upon which a lighter board with a very smooth top is hinged at one end. By means of the hinges and a side clamp, the upper board may be fixed at any desired angle to the base board. A small block of wood

with four smooth sides is made to slide on the upper board. (Two of these sides have twice the area of the other two). It is pulled along by a weight, hung from a thread which passes over a universal pulley clamped at the end of the table.

The friction between the two surfaces under the different conditions mentioned above and the corresponding pressures between them are observed, and the coefficient of friction calculated in each case.

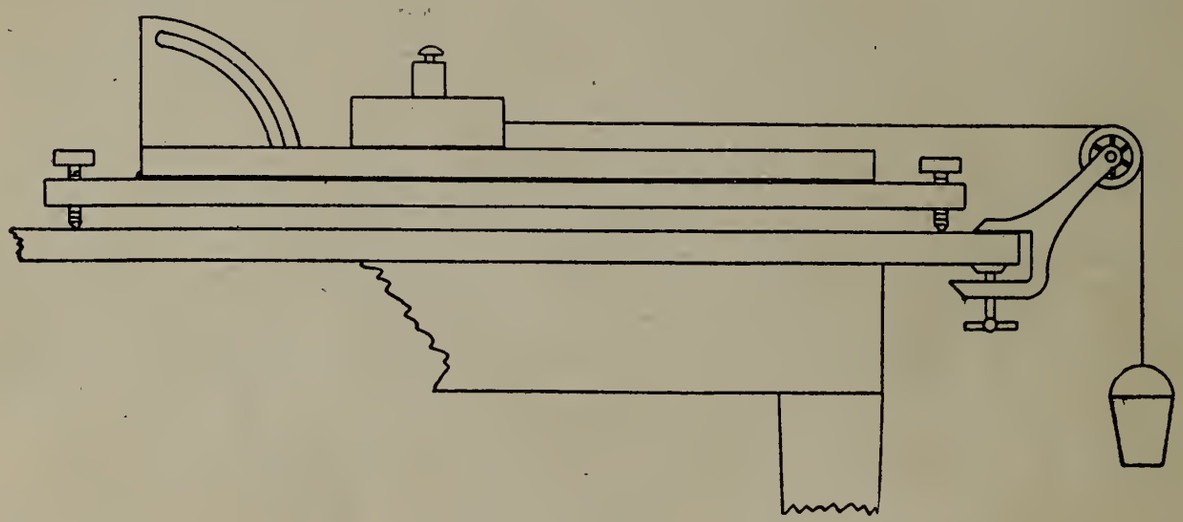


Fig. 19

DIRECTIONS.—*Caution.*—Do not touch the friction surfaces with the hands. The oil of the skin will lubricate the spots touched and spoil the uniformity of the surfaces.

Place the top board flat upon the base and get its upper surface level. To do this place the level in the center with its length across the board, and adjust the two leveling screws at one end of the board until the bubble of air in the level is at the center mark. Then place the spirit level lengthwise of the board and adjust the third leveling screw until the bubble stays at the center mark. (In every case below, adjust the pulley until the thread is parallel to the upper board.)

Static and kinetic friction.—Fasten one end of the thread to the hook on the block, pass it over the pulley, and fasten

the other end to the bucket. Draw the block to the far end of the friction table with its flat side on the board, and place a 100 gram mass on top of it. Then carefully place shot in the bucket until the block just begins to slide. The observer should hold one hand under the bucket while dropping in the shot, to prevent spilling the latter if the bucket begins to drop suddenly. Owing to the fact that the friction table is not uniformly smooth all over its surface, or that the relative directions of the grain of block and board are not exactly the same in successive trials, or to other causes, the amount of shot needed may vary in successive trials. Do not spend too much time in trying to get such trials to agree. Repeat the trial three or four times and take the mean weight of bucket and added shot for the force necessary to overcome the friction.

Under the same conditions vary the amount of shot in the bucket and find what weight of bucket and shot (the mean of several trials) is necessary to keep the block moving uniformly after a slight push has been given to start it. Find the weight of block and added 100 grams mass, and from the data, calculate the coefficients of static and of kinetic friction and compare them.

Friction corresponding to different areas of contact.—Turn the block on edge and adjust the pulley until the thread is parallel to the upper board. Place the 100 gram mass on the block and find as before what weight is necessary to keep it in uniform motion after a start has been given it. Measure the areas of the two surfaces of the block that were used as friction surfaces. Compare the kinetic friction in this trial with that in the previous trial and note what effect the area of the surface of contact has upon the friction.

Variation of friction force F with total normal pressure N.—Find the friction force when the block is laid flat with

100 grams mass on it. Repeat with 300 grams mass on it. Show from your data that F is directly proportional to N. (Remember that N includes the weight of the block in each case). Compare the coefficients of kinetic friction in the two cases.

Variation of friction force with nature of surface of contact.—Place the strip of aluminum (or glass) on top of the friction board, holding the former by the edges while handling it. Determine the mass necessary to slide the block with 100 grams on it uniformly along the surface of the metal. Calculate the coefficient of friction and compare it with that between wood and wood.

To determine the critical angle.—Remove the aluminum plate, and take the thread from the block. Tilt the top board to such an angle that when started by a slight push, the block will slide *uniformly* down the board. Clamp the board in this position and measure the perpendicular "*a*" dropped from the lower face of the upper end of the board upon the base, and the distance "*b*" from the hinged end of the board to this perpendicular. See Fig. 20. The ratio

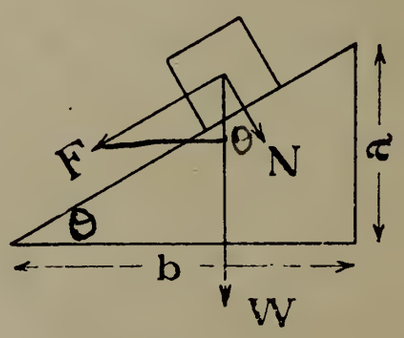


Fig. 20

a/b is equal to the tangent of the critical angle. Compare this tangent with the coefficient of friction ϕ found for the flat block and 100 grams mass sliding on the horizontal wooden surface with uniform motion.

Question.—Prove with the aid of a diagram the fact that tangent $i = \phi$, and show that "i" is independent of the mass of the block.

12 STRETCHING WIRES AND YOUNG'S MODULUS

To study laws of stretching wires and to determine Young's modulus of elasticity. (D. 164-172, G. 20, 87. K. 235-239, 244, W. 122, 172.)

Apparatus.—Stretching wire apparatus with optical lever, telescope and scale, four 2Kg. and two 1Kg. weights, micrometer calipers, beam compass, meter stick, weight holder, and three wires, (two of the same diameter but different lengths, and a third of the same length as one of these but having a different diameter.)

Theory, Description and Method:—*The elasticity of a body is its tendency to recover from distortion when the forces which produce the strain are removed.* By strain we mean the change of shape or volume produced by the forces. By stress we mean the internal forces between contiguous parts of the body when it is in a state of strain. Hook's law states that *within the elastic limits, stress is proportional to strain. The measure or modulus of elasticity is the ratio of the magnitude of the stress to that of the accompanying strain.*

In the case of simple longitudinal strain, as for instance, a stretching wire, this modulus is known as Young's modulus. In this case the stress is equal to the external force per unit area of cross section of wire, or $= F/a$ where F is the force and a the area. The strain or stretch is the extension per unit length of wire, or $= l/L$ where l is the total extension and L the length. Then for a circular wire of radius r, the area is πr^2 and Young modulus is

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{a} \div \frac{l}{L} = \frac{FL}{\pi r^2 l}$$

The apparatus consists of a supporting frame of two heavy steel rods about 150 cm. long, mounted vertically on a tripod with leveling screws, and fitted with a clamp at the

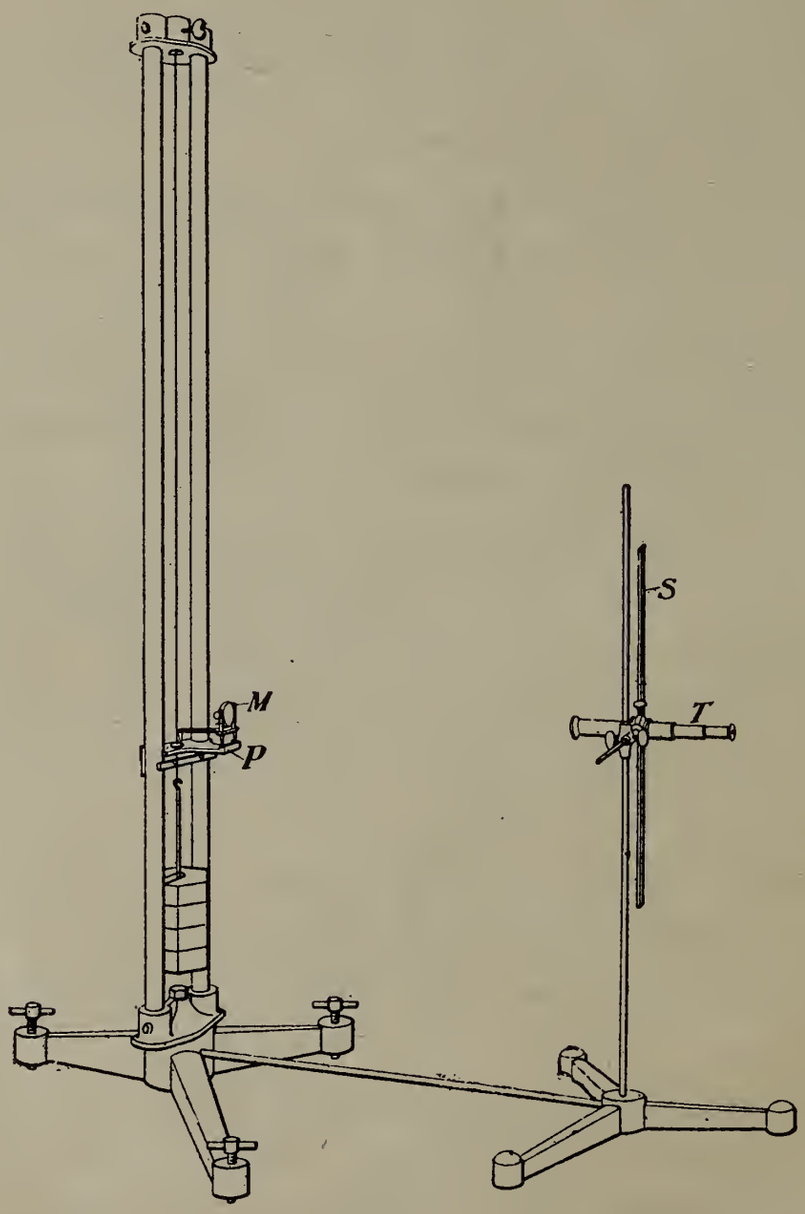


Fig. 21

top for holding the wire to be tested. See Fig. 21. A small bushing is clamped on the wire near its lower end, and an adjustable platform, sliding on the supporting rods, may be clamped at the same height as this bushing. The wire is

stretched by means of weights which are placed on the support hung from its lower end, and the amount of the stretch is measured by means of an optical lever and telescope with vertical scale attached. The optical lever consists of a T shaped base with a leg at each extremity and a small plane mirror mounted on a horizontal axis above the short arm of the T and facing away from the third leg. This leg rests on the bushing clamped on the wire. The other legs rest on the platform. In its normal position the plane of the

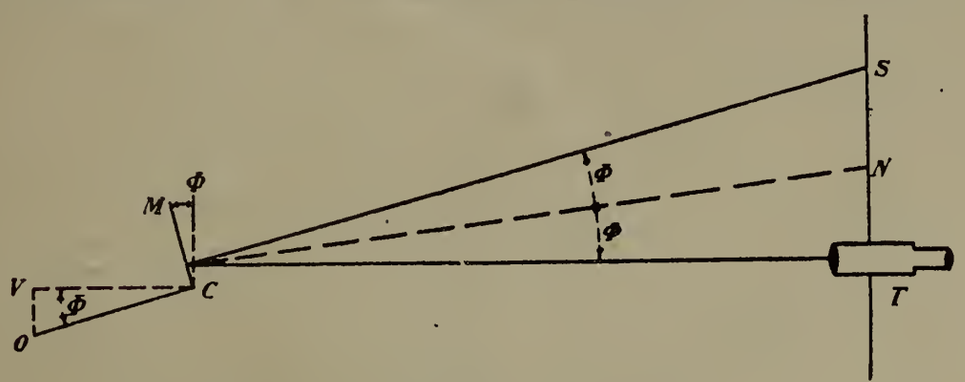


Fig. 22

mirror is vertical and the telescope is mounted directly in front of it at a distance of about 80 cm., so that on looking through the telescope when it is properly adjusted, we see the center of the scale reflected in the mirror. If now the wire is stretched, the leg V is lowered, (see Fig. 22) tilting the lever through an angle ϕ. The mirror and its normal $N\,M$ will also be tilted through this angle ϕ. (In the figure M is misplaced from the intersection of lines from S, N and T.) But by the law of reflection of light rays, the angle made by the incident ray (from scale S to mirror M) with the normal to the mirror (MN) is equal to the angle between the normal and the reflected ray (from the mirror M to the telescope T), and by the geometry of the figure we see that these angles equal ϕ also. The point S on the scale will therefore be seen in the telescope. For the small angles of tilt to be used, the point of intersection N between

normal and scale may be considered as halfway between S and T, and NM as equal in length to TM. Then in the similar triangles VCO and NMT we have

$$\frac{VO}{VC} = \frac{TN}{TM}, \text{ or } VO = \frac{VC \times TN}{TM}$$

where VO is the stretch of the wire, VC is the perpendicular distance from the leg V to the line joining the other two legs, TN is half the distance between the points on the scale observed in the telescope before and after tilting the mirror, and TM is the distance from scale to mirror.

DIRECTIONS.—*Caution.*—In order to prevent bending and kinking of the wires, keep them stretched by a small mass. The wire when not in use should be hung on the supports provided.

Preliminary.—Place the optical lever on a sheet of paper and press lightly on it, leaving an imprint of the three legs. Draw a straight line between the two nearer marks and drop a perpendicular from the third mark upon this line, and measure its length. Measure also the diameters of the three wires with the micrometer calipers, and the effective stretching length between bushing and lug with the beam compass and meter stick. After the stretching weight is removed, the wire to be used is slipped through the hole in the platform and fastened in the upper clamp by means of the lug. *To free the wire of all bends an initial stretching weight of 2Kg. is hung on it, and all no load readings are to be taken with this weight attached.* The platform is placed in line with the bushing on the wire, and the tripod is leveled so that the bushing slips freely through the opening when the wire is stretched. The optical lever is then placed on bushing and platform. The telescope and platform must be adjusted until the middle of the scale is

seen in the former. The telescope has three tubes which slide within each other, one containing the object lens, the second a pair of cross hairs, and the third the eyepiece. The latter must be drawn out until the cross hairs are seen sharply focussed; then the second tube is drawn out of the first until there is no parallax between the cross hairs and the scale divisions seen in the telescope; i. e. until there is no relative motion between the two when the eye is moved from side to side in front of the eyepiece. This adjustment should not be disturbed during the remainder of the experiment. The distance from mirror to center of scale should then be measured.

Relation between stretch and length of wire.—Suspend the shorter wire with a weight of 5Kg. and note the reading at the cross hairs in the telescope. Then carefully remove three kilograms, the stretching weight, and note the no load reading in the telescope. Replace the wire by a longer one of the same diameter, and note the no load and load readings for the same stretching force. Then calculate the stretch by the formula given, and the ratio of stretch to original length for both trials, and compare them.

Relation between stretch and cross sectional area of wire.—Replace the last wire used by the third one, having the same length but a different diameter, and take the load and no load readings with the same stretching force as before. Calculate the stretch and the cross sectional areas of the last two wires used. Calculate also the product of stretch and area for each wire, and compare these products.

Relation between stretch and stretching force, Hook's law—Using the same wire as in the last case, change the stretching force to 5Kg. and note the load and no load readings. Calculate the stretch as before. Calculate also the ratio of

the stretch to the stretching mass for each of the two trials with this last wire, and compare these ratios.

Young's modulus.—From the data taken in the last case calculate the value of Young's modulus for the wire used (brass or steel) Assume $g = 980$ cm/sec^2.

QUESTION.—The data taken show that the stretch $l = CmL/A$, where m is the mass producing the stretch, and C is a constant. What is the value of this constant?

13. LAWS OF BENDING OF RODS.

MODULUS OF ELASTICITY.

To study the relation between the bend of a rod and its length, breadth, depth, and material, and the bending force; also to calculate Young's modulus for brass and steel. (D. 164-173; G. 20, 88; K. 235-239, 244; W. 122, 173.)

APPARATUS.—Channel iron bench about a meter long, micrometer screw, micrometer calipers, two blocks with knife edges, two weights, weight holder and loop of thread, meter stick, and three rods, two of them having the same square cross sections but one of brass, the other of steel, and the third of steel with a rectangular cross section.

THEORY, DESCRIPTION AND METHOD.—(Read that part of the theory in experiment 12 which deals with elasticity.) It has been found that when a rod is bent out of its original position, the amount of the bend B is directly proportional to the cube of its length L and to the force F producing the bend, and inversely proportional to the cube of its depth d and to its breadth b. That is

$$B = C\frac{FL^3}{bd^3}$$

where C is a constant depending upon the mode of support and the material of the rod. If the rod be supported at both ends and the force be applied midway between them, this constant $C = 1/(4Y)$ where Y is the modulus of elasticity of the material.

In this case it is readily seen that the lower layers of the rod are stretched and the upper layers compressed. The modulus of elasticity involved is therefore Young's modulus. The relation between bend and modulus is

$$B = \frac{FL^3}{4Ybd^3}, \text{ or } Y = \frac{FL^3}{4Bbd^3}$$

To study the laws of bending a comparison is made between the values of the bend obtained for different values of one of the factors upon which it depends, all the other factors remaining constant. The relation between the bend and this factor can then be determined. If for instance we determine the amount of bend corresponding to two or three different values of the length when all other factors are kept constant, and find on examination that the ratio between corresponding values of bend and length cubed is constant, it shows that the bend is directly proportional to the cube of the length, etc. For these observations we use an iron bench at the middle of which is a vertical support for a micrometer screw. See Fig. 23. Two knife edges placed across

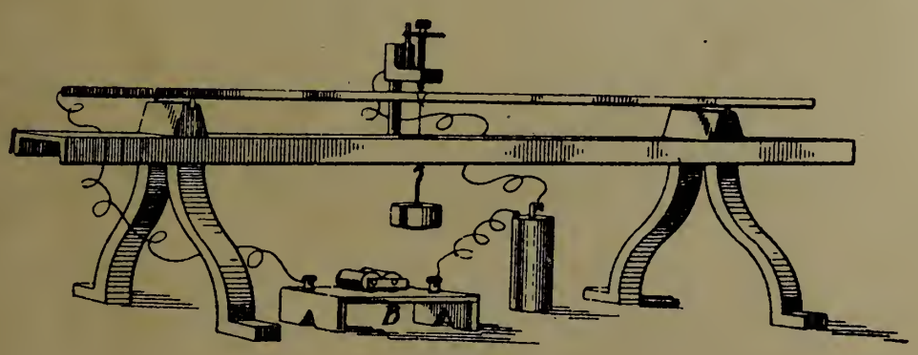


Fig. 23

the bench, equidistant from the screw, support the rod to be used. The micrometer is adjusted until its tip just touches the rod, first when the latter is unloaded except for the wire frame, then when it is loaded with a known mass placed on the frame. A reading of the micrometer is taken in each of these positions, and the difference between the two readings gives the bend of the rod.

The micrometer screw has a pitch of a half millimeter. Its straight scale is divided into millimeters, its circular scale into 100 equal divisions so that each division represents 1/100 of 1/2 millimeter or 1/2000 cm. To avoid errors in reading the micrometer the following method may be used: Adjust the micrometer until the screw tip is about 1/8 inch above the rod, the zero of the disc coinciding with one edge of the vertical scale. Carefully turn the screw until the tip just touches the rod, counting the number of complete revolutions and observing the fractional part of a revolution (the number of divisions beyond zero through which the disc is turned), estimating to 1/10 division. Thus for two complete turns and 10 and 1/10 divisions beyond zero the reading would be 210.1 *divisions*. Call this reading R_0. Return the screw to its original position, suspend a mass from the rod, and proceed as before taking the new reading R_1 when the screw just touches the rod. The bend in *cm.* will be $(R_1 - R_0)$ /2000.

It is somewhat difficult to see just when the point of the screw comes in contact with the rod, therefore an electric buzzer B is used as a detector. See Fig. 23. A dry cell is connected by means of wires to the screw and to the buzzer and from the latter to one end of the rod. Hence when the screw comes in contact with the rod, the circuit will be closed and the current of electricity from the cell flowing through the buzzer will cause it to emit a humming noise.

The tip of the screw and the part of the rod which it touches must be kept well polished to offer as little resistance to the flow of current as possible, otherwise the detector will not buzz until there is considerable pressure between screw and rod.

Directions.—*Relation between bend and length of rod, and between bend and force producing the bend.*—Polish the middle of the rods where the screw tip will touch them. Place the knife edges 80 cm. apart with the micrometer half way between them and lay the small steel rod flat on them. Lay the loop of thread over the middle of the rod, bring both ends down through the hole in the bench, and hook them together with the wire frame which is to support the "weights." Adjust the micrometer and have the threads about 5mm. apart so that the screw can touch the rod between them. Turn the screw until its tip just meets its image reflected from the rod. The humming of the buzzer will indicate when this occurs, if it cannot be seen with the eye. Read the micrometer to 1/10 division and repeat twice more, averaging the three readings to get R_o. Then hang the smaller mass on the wire frame, take an average of three readings for R_1, and calculate the bend for the weight of this mass, m_1. In the same way find the bend corresponding to the weight of a larger mass, m_2. Then move the knife edges closer until they are 60 cm. apart, the micrometer still being midway between them, and find the bend under the weight of the larger mass. From these data show that the bend varies directly as the force applied and as the cube of the length between supports by calculating the ratios of corresponding values of bend and mass producing the bend for each case tried, and comparing them; and of bend and length cubed for each case and comparing them.

Relation between bend and breadth of rod, and between bend and depth of rod.—Measure with the micrometer calipers the width and thickness of all three rods. Take several measurements at different parts of the rod and average them. Using a length of 80 cm. and the larger mass determine the bend of the larger steel rod, first with its flat side (the breadth) on the knife edges, then with its narrow side, (which was the depth in the previous trial) on knife edges. If time permits, redetermine the bend of the small steel rod for the same length and mass; if not use the data already obtained for this case. Show with your data that the bend varies inversely as the breadth and inversely as the cube of the depth of the rod, by calculating the products of corresponding values of bend and breadth for each case tried, and comparing them; and of bend and depth cubed for each case and comparing them.

Variation of bend with material of rod. Young's modulus.—Determine the bend of the brass rod for a length of 80 cm. and the weight of the small mass. Compare this with the bend of the small steel rod under the same conditions and assuming that they have the same breadth and depth, find the ratio of the bends, that of steel being taken as 1.

From the data for the first and last trials calculate Young's modulus for steel and for brass.

Assume $g = 980$ cm. /sec^2.

QUESTION.—Using the formula given for the bend of a rod, what is the simplest expression for the ratio of the bend of one rod to that of another? (Distinguish between the various factors for the two rods by means of subscripts 1 and 2.)

14. LAWS OF TWISTING OF RODS

Modulus of Rigidity.

To study the relation between the amount of twist of a rod and its length, its diameter, the force producing the twist, and the nature of the rod; also to find the modulus of rigidity of steel and of brass. (D. 164-170; G. 20, 89; K. 235-239, 243; W. 122, 171, 174.)

Apparatus.—Iron bench, micrometer calipers, two weights, meter stick, and four rods, three steel ones of different diameters, and one brass one.

Theory, Method and Description.—(See that part of theory in experiment 12 which deals with elasticity.) It has been found that the amount of twist in a metal rod is directly proportional to its length and to the force producing the twist and inversely proportional to the fourth power of its radius. The rigidity of a rod under the action of a certain force is inversely proportional to the twist produced by the force. That is if R and T are respectively the rigidity and the angle through which the rod is twisted, the relative rigidity of two rods will be $R_1/R_2 = T_2/T_1$.

When a rod is twisted the distortion is a change of shape only; that is, it has undergone a shear. The ratio of the shearing stress per unit area to the shearing strain per unit length is the "modulus of rigidity" (sometimes called coefficient of torsional rigidity or simple rigidity.) It may be shown that this modulus is

$$n = \frac{2LM}{\pi \theta r^4}$$

where L is the length of the rod, r its radius, M the moment of the torsional couple, and θ the angle (in radians) through

which the rod is twisted. If the rod is twisted by a mass m hanging with a moment arm R, we have $M = mgR$ and

$$n = \frac{2LmgR}{\pi\theta r^4}$$

To study the relation between the twist of a rod and any of the factors upon which the twist depends, the twist for different values of this factor are obtained, all other factors remaining constant. From a comparison between a value of the factor and corresponding value of the twist, the law of variation between them can be determined. Keeping all factors constant but using two rods of different materials and observing the twist for a given force, their relative rigidity can be determined by calculating the inverse ratio of their twists.

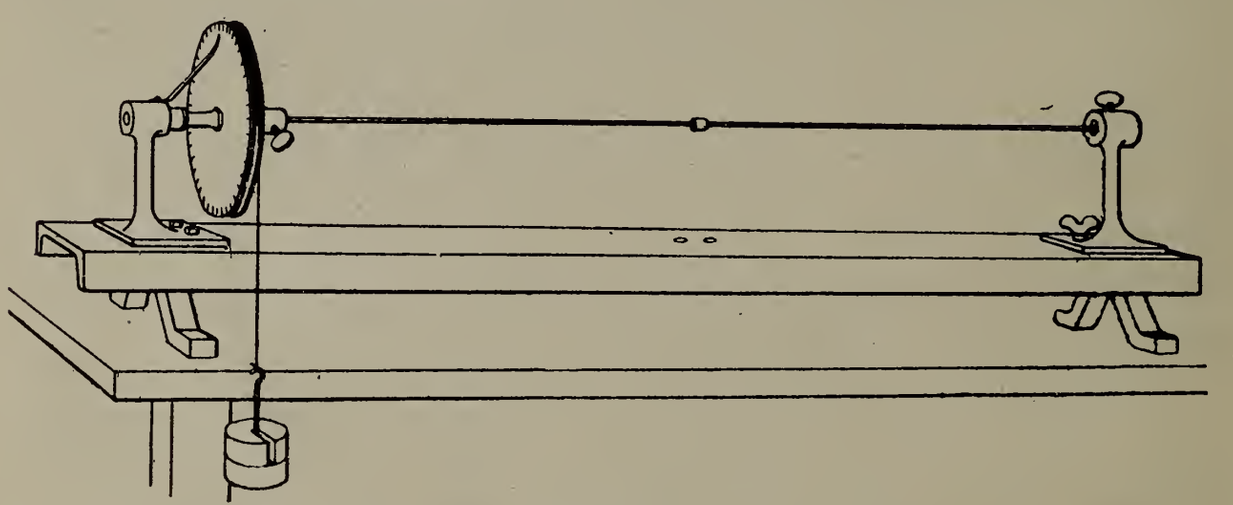


Fig. 24

The torsion bench is about a meter long. See Fig. 24. An iron clamp may be screwed fast at various positions on the bench and serves to hold in a fixed position the rod to be twisted. The end of the rod is held by a clutch mounted on ball bearings at one end of the bench. Fastened to the clutch is a grooved wooden wheel from the circumference of which is to be suspended the mass which produces the twist.

A circular scale is attached to the wheel and a fixed pointer indicates on the scale the angle through which the rod has been twisted.

Directions.—*Relation of angle of twist to length and to force producing twist.*—Clamp the smallest steel rod in position for twisting, having distance between the clamps about one meter. (Some of the small steel rods are copper plated and so do not look like steel.) Take the initial or no load reading T_0 to 0.1 division on the scale when only supporting cord and hook hang from the wheel. Then suspend the smaller mass from the wheel and note the load reading T_1. As a check against possible slipping the no load reading should always be repeated after a load reading has been taken. The difference between these readings gives the angle of twist, $T°$. Replace the small weight by the larger one and find the twist produced. Then change the position of the movable clamp so that the rod is about half its original length and find the twist produced by the larger mass. Note the weight of the masses and measure the length of the rod between the two lugs in each case and from the greater length subtract the length of the middle lug (since the part of the rod within this lug is not appreciably twisted.) From the data taken, show how the twist varies with the length and with the weight which produces the twist, by comparing the ratios of corresponding values of twist and length in each case, and the ratios of corresponding values of twist and twisting mass in each case.

Relation between twist and radius of rod.—Determine the twist for the next larger rod, using a length of about one meter and the larger mass. Do the same for the largest steel rod under the same conditions. Measure the mean diameters of the two rods with the micrometer calipers, and from your data show the relation between the twist and the

radius of the rod, by comparing the products of corresponding values of twist and the fourth power of the radius in each case.

Relative rigidity of steel and brass.—Determine the twist of about one meter of brass rod for the larger mass. If time permits redetermine the twist of the largest steel rod under the same conditions; if not, use the previous determination, and calling the rigidity of the steel rod unity, calculate the relative rigidity of the brass rod.

Modulus of rigidity.—Measure the circumference of the grooved wheel with a thread and meter stick, and calculate its radius R. Reduce the angle of twist in the last two observations from degrees to radians. Then from data taken calculate the modulus of rigidity for steel and brass. Assume $g = 980$ cm./sec^2.

QUESTION.—If one rod has a times the length and $1/b$ times the diameter of a second rod, and c times its modulus of rigidity, what is the ratio of the twist of the first to that of the second when the masses acting with the same moment arm for both, are in the ratio d to f respectively?

15. MOMENT OF INERTIA.

To find the moment of inertia of a rectangular bar of steel. (D. 81-92; G. 89 A; K. 135-140; W. 85.)

APPARATUS.—Upright support with two pulleys, rotating disk, rectangular bar, two masses (45 and 100 g.) coarse balance and weights, beam compass, two meter sticks, and string.

THEORY, METHOD, AND DESCRIPTION.—*The moment of inertia of a body about an axis is the sum of the products of the mass of each particle of the body into the square of its*

distance from the axis of rotation. If I be the moment of inertia, m_1, m_2, etc., the masses of different particles and r_1, r_2, etc., their respective distances from the axis of rotation, then

$$I = m_1 r_1^2 + m_2 r_2^2 + etc.$$

In the equations of motion for a body in rotation its moment of inertia is the equivalent of its mass in the equations for translation. For instance, the kinetic energy of a mass m with linear velocity v is $mv^2/2$. Its kinetic energy of rotation with an angular velocity ω is $I\omega^2/2$.

If the whole mass of the rotating body be considered concentrated at a distance k from the axis of rotation, such that its kinetic energy is unaltered, since the linear velocity at this distance is $v = k\omega$, we may write

$$mv^2/2 = mk^2\omega^2/2 = I\omega^2/2$$

Hence

$$I = mk^2$$

and k is called the radius of gyration of the body.

It may be shown that when a body of mass m has a moment of inertia I_o about an axis through its center of mass, its moment of inertia about a parallel axis at a distance d from the first is

$$I = I_o + md^2$$

The method of finding the moment of inertia of the bar is to set it in rotation by a known falling mass. Its kinetic energy of rotation is then the difference between the potential energy of the suspended mass and its kinetic energy at the end of its drop. The angular velocity having been determined, the moment of inertia is calculated from the kinetic energy of rotation.

The apparatus consists of an aluminum disk mounted on a vertical spindle, and a vertical support upon which are fixed two pulleys. See Fig. 25. A known mass is suspended by means of a cord which passes over the upper pulley and under the lower one, and is wound several times around a wooden spool on the vertical spindle, so that when the mass drops, it sets the disk in rotation. The metal bar whose moment of inertia is to be found is placed on the disk and held in position by projecting pins which fit into holes

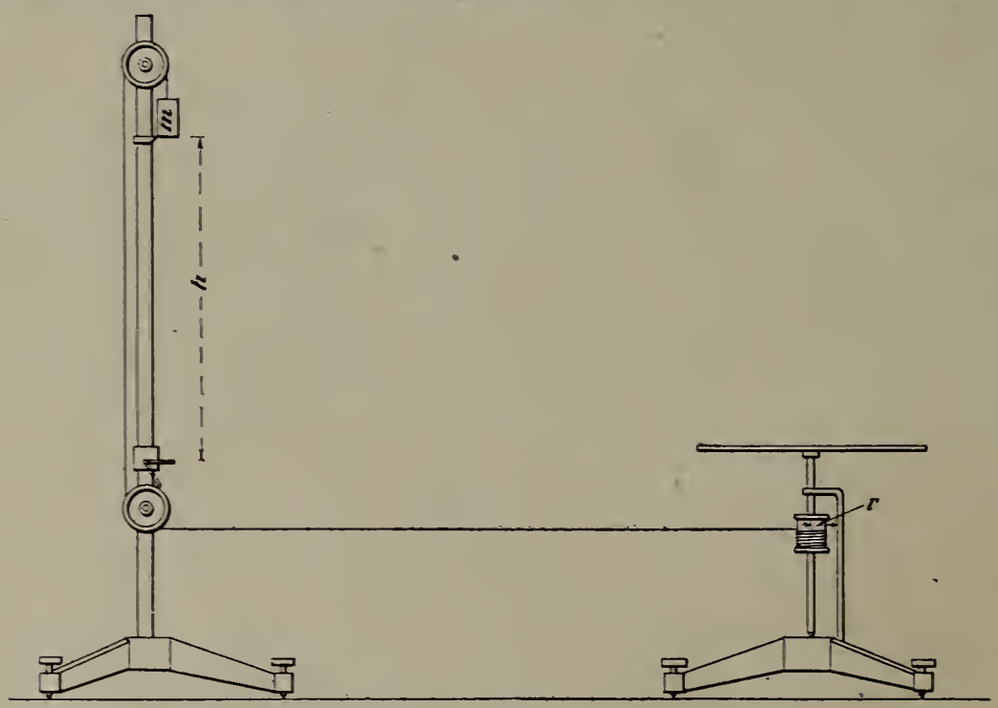


Fig. 25

in the disk. The disk is released by tilting a small lever which catches on a projecting pin on the wooden spool. A small adjustable platform on the upright support serves to locate the height to which the mass has dropped after falling a given number of seconds.

As the mass falls its potential energy is converted into kinetic energy, and at any instant its loss in potential energy, is equal to its gain in kinetic energy plus that acquired by the rotating disc. If m be the falling mass, h the distance it falls in time t, g the acceleration of gravity, v the final

velocity of m at the end of its fall, ω the angular velocity of the disk corresponding to v, and I the moment of inertia of the rotating disk, spindle, and spool,

$$m\ g\ h = \tfrac{1}{2}\ m\ v^2 + \tfrac{1}{2}\ I\ \omega^2$$

If we assume the friction of the apparatus to be constant the mass will fall with constant acceleration a and the distance fallen will be

$$h = at^2/2$$

The velocity at the end of the fall will be

$$v = at.$$

Hence from the preceding equation

$$v = 2h/t.$$

The linear velocity of the circumference of the spool will at any instant be the same as that of the falling mass since they are connected by a cord, and as the angular velocity of the spool equals the linear velocity of its circumference divided by its radius r, the angular velocity at the end of the fall will be

$$\omega = v/r.$$

The linear and angular velocities can therefore be determined from measurements of m, h, t and r and the value of I may be calculated from the equation of energy.

When the bar is placed on the disk and the latter is set in rotation by the larger mass m_1, the distance fallen h_1, in the same time t, will be different, and therefore the velocities v_1 and ω_1 will be different. If I_1 be the moment of inertia of the bar, the equation of energy becomes

$$m_1 g h_1 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} (I + I_1) \omega_1^2.$$

h_1, v_1, and ω_1 can be determined as before and I being known, I_1 can be calculated from the equation.

It may be shown that the moment of inertia of a rectangular bar of length "l" width "b" and mass "m" is

$$I_1 = m (l^2 + b^2) /12.$$

I_1 can be calculated from this equation as a check on the above determination.

DIRECTIONS.—Loop the thread over the projecting pin on the spool and wrap it several times around the spool; then pass it under and over the pulleys as shown in the diagram and suspend the mass of 45 grams from the other end. Adjust the apparatus until the bottom of the mass hangs level with the mark on the support just below the upper pulley, and catch the pin on the lever so that the disk does not rotate. Note the ticks of the seconds clock and when all is ready tip the lever and release the disk on the tick of the clock sounder. Watch the falling mass and at the same time count the ticks of the sounder, beginning with zero at the instant of release. Observe the point reached by the mass on say the seventh tick, or at the end of the seventh second after beginning to fall. Repeat, and adjust the movable platform until the mass just strikes the platform on the seventh tick. Measure with beam compass and meter sticks the height h through which the mass falls in seven seconds. Find also the radius of the spool by allowing the cord to unwind slowly from the spool and measuring the distance moved through by the suspended mass when the spool makes five complete revolutions. This distance will be 10 π times the radius of the spool. Calculate v and ω and from the equation of energy obtain the value of I, as-

suming $g = 980$ cm/sec^2. Then place the steel bar on the disk, suspend the mass of 100 grams from the cord and repeat the above performance. From the data obtained and the equation of energy calculate the value of $(I+I_1)$ and subtract the value of I previously obtained.

Next measure the length and width of the bar, and find its mass on the coarse balance. From the equation given above calculate its moment of inertia and compare it with the value of I_1 obtained in the experiment.

Finally determine by about ten trials, through what distance you can raise and lower the platform on the support without being able to detect any difference between the simultaneous click of the clock sounder and that of the faling mass on the platform. A table of formulas for various moments of inertia will be found in the appendix.

Question:—a) What would be the percent error in your determination of I if the center of mass of the bar were displaced one mm. from the axis of rotation of the disk?

What percent error is introduced in your result by the possible variation in the height h as determined in your final trials?

16. THE TORSION PENDULUM.

To show the relation between the period of a torsion pendulum and the length and radius of its suspension; to find the moment of inertia of a ring and of a cylinder by the torsion pendulum, and the modulus of rigidity of the suspending wire.

(D. 81-89, 118, 119, 115; G. 89A, 736; K. 125, 135-140; W. 85, 175.)

Apparatus.—Pendulum support, disk, ring, beam, compass, meter stick, coarse balance, weights, micrometer cal-

ipers, two cylinders, and three wires, two of the same length but different diameters, the third of a different length with the same diameter as one of the other two.

THEORY, METHOD, AND DESCRIPTION.—(See also theory of experiments 14 and 15.) The torsion pendulum consists of a uniform, vertical, straight wire clamped at one end and supporting a heavy body at the other end. If the wire be twisted through a small angle without displacing it, and then released, the body will pass its position of rest, owing to its kinetic energy and will perform angular vibrations. By Hook's law the restoring couple at any instant will be proportional to the twist of the wire, and the vibrations will be harmonic and isochronous. The period of the pendulum is the time required to make a complete vibration, or double swing; that is, the time which elapses between two successive passages of the pendulum in the same direction through some point of its path, such as its position of rest.

It may be shown that the period of such a pendulum is

$$T = 2\pi\sqrt{I / M_1}$$

where I is its moment of inertia and M_1 the moment of the torsional couple when the wire is twisted through an angle of one radian. But M_1 depends upon the material, radius and length of the wire. In experiment 14 the value of the modulus of rigidity was given as $n = 2lM/(\pi r^4\theta) = 2lM_1/(\pi r^4)$ where l is the length and r the radius of the wire. From this we get $M_1 = \pi n r^4/2l$ and substituting this value of M_1 in our expression for the period, we have

$$T = 2\pi\sqrt{I\,l\,(2/\pi n r^4)} = 2\pi\sqrt{I\,l\,C}$$

where $1/C$ is a constant for a given wire (the constant of torsion).

From this equation we see that the period is directly proportional to the square root of the moment of inertia of the body, and to the square root of the length of the wire, and inversely as the square of its radius.

By comparing the periods of two pendulums of the same wire with the same body suspended but having different lengths, we can show that the second of these statements is true; by comparing one of these periods with that of a pendulum of the same length and suspended mass, but with a wire of different diameter we may show that the third statement is true. The first statement offers a means of determining the moment of inertia of the body.

To do this we use a pendulum consisting of a heavy frame similar to that in Fig. 21, at the top of which is clamped a wire supporting a heavy iron disk. A pointed stop may be pressed into contact with the center of the disk and prevents oscillations other than those about this center. The period of this pendulum is observed and will be

$$T = 2\pi \sqrt{I / M_1}$$

where I and M_1 are respectively the moment of inertia of the pendulum and moment of the torsional couple for unit angle. The body whose moment of inertia is to be determined is then fastened to the pendulum disk so that it is symmetrically placed with respect to the wire and disk. The period of this pendulum is then determined and will be

$$T_1 = 2\pi \sqrt{(I + I_1) / M_1}$$

where I_1 is the moment of inertia of the body. From these two equations we get the proportion

$$T^2 : T_1^2 :: I : (I + I_1)$$

from which we calculate I_1 in terms of the known moment I and the two periods.

The moment of inertia of the disk about an axis perpendicular to its face is $\frac{1}{2} m r^2$ where m is its mass and r its radius.

We may also calculate I_1 from the dimensions of the body as a check on the value obtained above. For instance if the body is a ring of radii r_1, and r_2 and has a mass m,

$$I_1 = m\ (r_1^2 + r_2^2)/2$$

(See Appendix for formulae of moments of inertia.)

From the expression for the period of the pendulum we may determine the modulus of rigidity of the suspending wire, when the other factors are known.

DIRECTIONS:—*Preliminary.*—Measure the mass of disk and ring on the coarse balance, and their radii with the meter stick. Measure also the radii of the wires with the micrometer calipers. Their lengths are best measured with beam compass and meter stick when they are suspended and stretched.

Moment of inertia of ring.—Pass the shorter wire through the disk and clamp it in the support. Measure its length. Press the stop into contact with the center of the disk, displace the latter through a small angle, and allow it to oscillate. When the oscillations are quite steady release the stop, and count the time for ten *complete* oscillations, by noting the passage of the index mark on the disk past the pointer on the frame. Repeat as a check and calculate the period of the pendulum. Unclamp the wire, slip the ring over it and replace it. The ring must be placed symmetrically on the disk with respect to its axis. Determine the period of this pendulum as before. Calculate the moment of inertia of the disk from its mass and radius and that of the ring from the periods and moment of inertia of the disk. Calculate its moment of inertia from its mass and radii also. Then compare these two moments of inertia for the ring.

Moment of inertia of two equal cylinders about an axis parallel to their own.—Two cylinders of equal mass and dimensions are placed on the disk of the pendulum at opposite ends of a diameter. If I be the moment of inertia of the disk alone about an axis through its center (the suspending wire), I_1 the moment of inertia of one cylinder about its own axis and I_2 the moment of inertia of one cylinder about the axis of the disk we may determine I from the dimensions of the disk $\frac{1}{2}\ mr^2$. Then by determining the period T of the pendulum alone and the period T_1 of the pendulum and two cylinders we have $T^2 : T_1^2 = I : I + 2I_2$. Hence we may calculate I_2. But if m_1 be the mass of one cylinder and r_2 the distance between its axis and the axis of rotation we have

$$I_1 = I_2 - m_1 r_2^2$$

(See Exp. 15 Theory). Hence we may calculate I_1. As a check I_1 may be calculated by the formula

$$I_1 = \tfrac{1}{2}\ m_1 r_1^2$$

where m_1 is the mass and r_1 the radius of each cylinder. We have therefore to determine the period of the pendulum alone and that of pendulum and cylinders together, the mass of the disk and its radius, the mass of each cylinder and its radius and the distance between their axes in order to determine I_1.

Variation of period with length and with radius of suspension.—Remove the ring and replace the wire suspension by the longer one of the same diameter, and determine its period as before. Then replace this wire by the third one of the same length but of different diameter, and determine its period. Show how the period varies with the length of suspension by calculating the ratio of period to square root

of length in the two cases where the diameter and mass were the same but the length varied. Show how the period varies with radius of suspension by calculating the products of period and square of radius in the two cases where the length and mass were constant but the diameter varied.

Modulus of Rigidity of Steel (or Brass.)—From the first period determined calculate the modulus of rigidity of the suspension wire.

QUESTION.—Which of the measurements in this experiment must be made with the greatest care, and why?

Read 89-90

17. THE SPIRAL SPRING BALANCE

To calibrate a spiral spring balance; to test Hook's law for a spiral spring; and to determine the weight and specific gravity of several substances with the spiral spring balance. (D. 162, 168, 191. G. 27, 112, 118, 119, 122. K. 176, 182, 183. W. 127, 129).

Apparatus.—Spiral spring balance, set of weights, (1 to 5 grams) forceps, samples of lead, brass and aluminum, small beaker for distilled water, small beaker of kerosene or alcohol.

METHOD, THEORY AND DESCRIPTION.—*The specific gravity of a substance is the ratio of its density to that of some standard substance.* The standard usually employed is water at 4° *C.* Since the density of a substance is its mass per unit volume, it follows that the specific gravity of a substance may be defined as the ratio of its mass to that of an equal volume of water. The method employed is based upon Archimedes' principal which states that *a body immersed in a fluid is buoyed up by a force which is equal (and opposite) to the weight of the volume of the fluid which the body displaces.*

In the spiral spring balance to be used, we have a spiral spring to which is suspended a small pan to hold the substance under experiment. See Fig. 26. The spring *S*

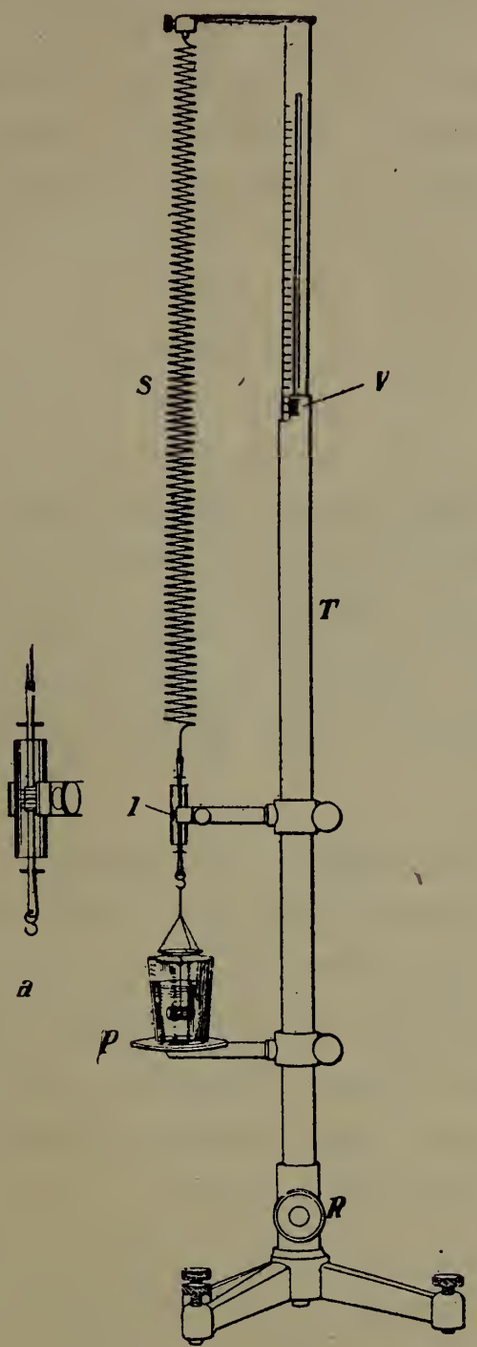


Fig. 26

is supported by two vertical telescoping tubes *T* mounted on a tripod base with leveling screws. It is suspended from the inner tube which is graduated in millimeters for about half a meter of its length; and can be moved up or down

by means of a chain rack and pinion R near the base. Between the spring and pan is an indicator I consisting of a small aluminum rod with two cross arms, one at either end of a short glass tube enclosing the rod and supported by an adjustable clamp on the outer tube. (See Fig. 26a.) The glass is whitened at the back, and has a fine black line passing entirely around its middle. The aluminum rod carries at its center a small cylinder bearing three equidistant black lines, the middle one of which is made to coincide with the line on the glass tube when the spring is brought to the zero position. A vernier V on the upper end of the tube, reads to tenths of a millimeter and gives the elongation of the spring under any load when the indicator is in the zero position. On the outer tube is also an adjustable support P for a vessel with liquid. According to Hook's law the stretch of the spring is (within limits) directly proportional to the force (weight) producing the stretch.

To determine the weight of an object with the balance, the latter must first be calibrated. That is, a curve must be drawn through points obtained by plotting a number of known weights and the corresponding stretch when each is placed in the pan. Upon observing the stretch produced by any object in the pan, the corresponding weight can be read off directly from the curve.

To determine the specific gravity of a solid (not soluble in water) we first determine its weight when suspended in air, W_1, then when it is completely immersed in water, W_2. The weight of an equal volume of water is then, by Archimedes' principle, equal to their difference $(W_1 - W_2)$ and the expression for the specific gravity of the body is

$$sp.\ gr. = W_1/\ (W_1 - W_2).$$

We are not interested however in the actual weight of the body, but in its ratio to that of an equal volume of water, and since the stretch of the spring is directly proportional to the weight producing it and we are here dealing only with ratios and differences of weights, we can make our calculations with the stretches themselves. That is since the weight in each case is some constant K times the corresponding stretch, $W = KS$ and

$$sp.\ gr. = \frac{KS_1}{KS_1 - KS_2} = \frac{S_1}{S_1 - S_2}$$

where S_1 and S_2 are the stretches corresponding to W_1 and W_2.

To determine the specific gravity of a liquid, a solid which has a greater density than the liquid is weighed first when suspended in air, then when completely immersed in water, and finally when immersed to the same depth in the liquid under experiment.

Directions.—*To draw the calibration curve and test Hook's law.*—Adjust the leveling screws and glass tube so that the indicator hangs in the center of the latter. Place the eye in the plane of the circle on the glass tube and turn the milled head which moves the inner tube until the middle mark of the indicator is in this plane. Observe the reading on the vernier and record this no load reading R_0.

Place a one gram weight in the pan (using the forceps to handle it) and turn the milled head until the indicator is in its zero position as before. Observe and record the load reading R.. Do the same for 2, 3, 4, and 5 grams, and then repeat the no load reading as a check.

Calculate the stretch $R - R_0$ for each weight. Plot the known weights as abscissae, and the corresponding stretches as ordinates and draw the calibration curve through the

points obtained. If this curve is a straight line the spring obeys Hook's law. (See introduction for method of plotting curves).

To find the weight of the samples of lead, brass, and aluminum.—Place each object in the pan and record the resulting load reading. Also record the no load reading after removing each object, and calculate the stretch produced by each. Refer the stretch to the calibration curve and note from the axis of abscissae the corresponding weight of the object.

To find the specific gravity of the objects.—Suspend about three inches of the fine wire from the pan and record the resulting reading R_0. Place the object in the pan and record the resulting reading R_1. Calculate the stretch $S_1 = R_1 - R_0$.

Remove the object, fill the beaker with *distilled* water and place it on the support. Adjust this support until the wire is immersed in the water except for about a centimeter of its upper end. Record the resulting reading R_2. To make the effect of surface tension as small as possible, have only a single strand of the wire passing through the surface of the water. Then fasten the wire securely to the object and immerse them to the same point on the wire. To avoid errors take care that no air bubbles adhere to the wire or to the sample when they are immersed in the water. Record the reading R_3 and calculate the corresponding stretch S_2. Since the effect of surface tension is included in both readings R_2 and R_3, it is eliminated from their difference S_2. From S_1 and S_2 calculate the corresponding specific gravity of the object. Do the same for each of the three samples.

To find the specific gravity of kerosene or alcohol.—Suspend the wire in the liquid as before and record the resulting reading R_4. Then suspend the brass sample at the same depth as it was in the water, and record the reading R_5. Calculate the corresponding stretch S_3. Note the previous

stretches for brass, S_1 and S_2 and calculate the specific gravity of the liquid from these three stretches.

Question (*a*).—Derive the expression for calculating the specific gravity of the liquid from the data taken. (*b*). Compare density and specific gravity of a body in the C.G.S. system; in the F.P.S. system.

18. SPECIFIC GRAVITY WITH HYDROMETERS.

(a) To determine the specific gravity of solids with a Nicholson's hydrometer. (b) To determine the specific gravity of a liquid with a Beaume hydrometer. (D. 162; G. 121-129; K. 182, 185; W. 147.)

Apparatus.—Nicholson's hydrometer, tall hydrometer jar, set known masses, two metal pieces, Beaume hydrometer, jars of solutions.

Description and Theory.—A Nicholson's hydrometer consists of a cylindrical metal vessel *B* at one end of which is attached a loaded chamber *D* (Fig. 27 b.) The latter serves to keep the vessel in an upright position when placed in a liquid. A slender stem *S*, carrying a small pan for holding objects, projects above the metal chamber.

Let M_1 be the mass which, when placed on the pan *P*, will cause the hydrometer to sink in water to a given mark *O* on the stem *S;* M_2 the mass in addition to the object, whose specific gravity is desired, to sink the vessel to the same mark; and M_3 the mass to produce the same result when the object *N* is placed in the pan *D* in the water. Then M_1-M_2 is the mass of the object in air and M_3-M_2 is the apparent loss of mass in water. Since the specific gravity is equal to

the mass of the body in air divided by the apparent loss of mass in water we have

$$\text{Specific gravity} = \frac{M_1 - M_2}{M_3 - M_2}$$

The Beaume hydrometer is of the variable immersion

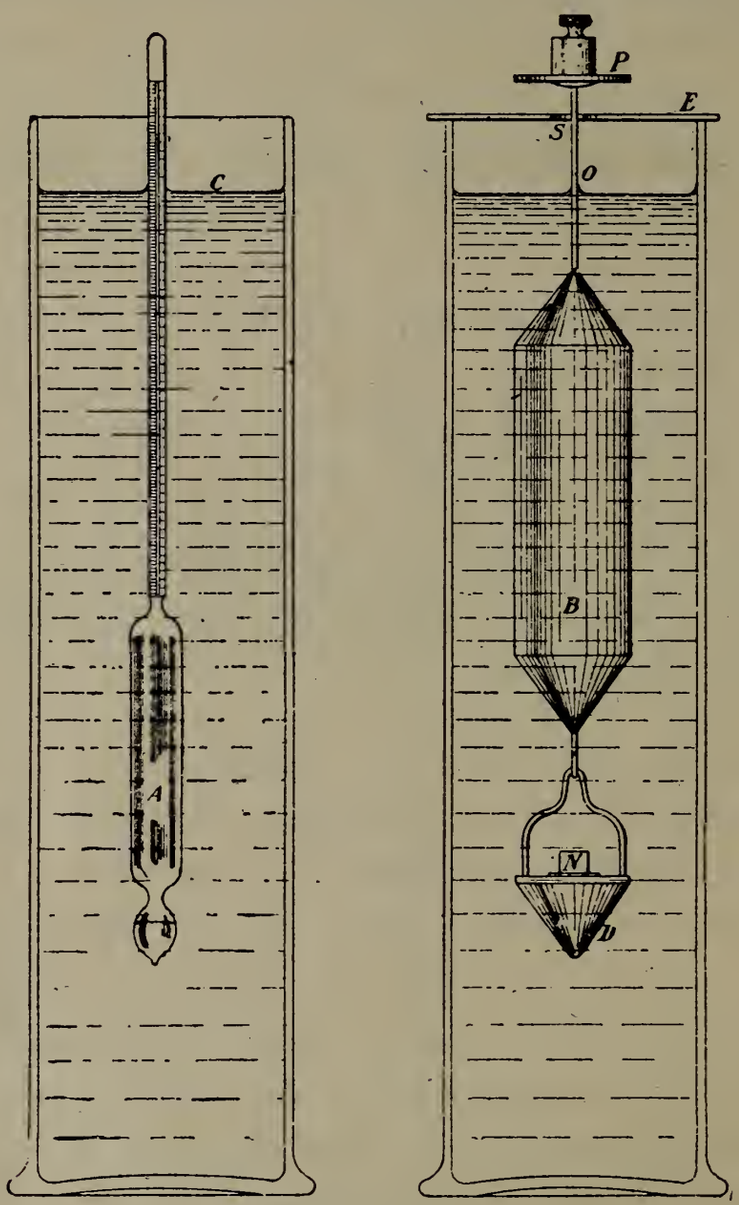


Fig. 27 a and b

type and consists of a glass bulb *A* (Fig. 27 a) with an elongated stem. The bulb is loaded at one end so as to make it float in an upright position. The elongated stem has a scale which has been graduated so that its reading at the

surface of the liquid in which the hydrometer has been placed gives the specific gravity of the liquid. Care must be taken in reading the hydrometer to avoid errors due to the distortion of the surface around the stem by capillary action. Thus in Fig. 27a the reading C is in line with the under surface of the liquid.

Directions.—*Determination of the specific gravity of a piece of metal with Nicholson's hydrometer.*—Pour distilled water into the hydrometer jar until there is enough to float the hydrometer. When placing the hydrometer into the jar see that no air bubbles cling to the vessel. Place the cardboard E around the stem in order to prevent the hydrometer from sinking if the mass added to the pan is too large. Place known masses upon the pan P till the mark on the stem S is just at the surface of the water and record the value of these masses as M_1. Remove the known masses from the pan and place in it one of the pieces of metal. Again add known masses till they, together with the piece of metal, sink the hydrometer to the mark on the stem. Call the value of the known masses in this case M_2. Put the piece of metal in the lower pan D and determine the mass M_3 which must be placed in the pan P to sink the hydrometer to the mark on the stem. From the data obtained calculate the specific gravity of the metal by means of the relation

$$\text{Specific gravity} = \frac{M_1 - M_2}{M_3 - M_2}$$

Repeat the above determination and make a second calculation of the specific gravity of the metal as a check on the first. In a similar manner make two determinations of the specific gravity of each of the metals furnished you.

Specific gravity with the Beaume hydrometer.—Liquids whose specific gravity are to be determined will be found in jars near the sink. Place the hydrometer in each solution and note the mark on the stem which is at the surface of the liquid. This mark *C*, Fig. 27a, can be read most advantageously by sighting along the under surface of the liquid through the walls of the jar. After removing the hydrometer from one solution, wash and dry thoroughly before placing it in another solution. Make at least three separate readings for the specific gravity of each liquid.

QUESTIONS.—1. What effect does capillarity have upon the reading for the specific gravity of a liquid with the Beaume hydrometer? 2. In the determination of the specific gravity of a metal with Nicholson's hydrometer what will be the effect on the result of trapping a bubble of air under the metal object when it is placed in the pan *D*?

19. SPECIFIC GRAVITY BY HARE'S METHOD.

To determine the specific gravity of liquids. (K. 174, 182-185; W. 144.)

APPARATUS.—Hare's apparatus, beakers, distilled water, meter stick, measuring rod, liquids.

DESCRIPTION AND THEORY.—The Hare's apparatus used for finding the specific gravity of liquids consists of three long glass tubes fastened in a vertical position on a common base. The lower ends of the tubes dip into the liquids whose specific gravity is to be determined and the upper ends are joined together. If the air is drawn out from the tubes the liquids will rise in them due to the difference between the

pressures on the inside and the outside. The pressure on the outside is the same for each tube, namely one atmosphere, and since the tubes communicate with each other, the pressure on the inside must be the same. The pressure on the inside is made up of the pressure of the column of liquid and

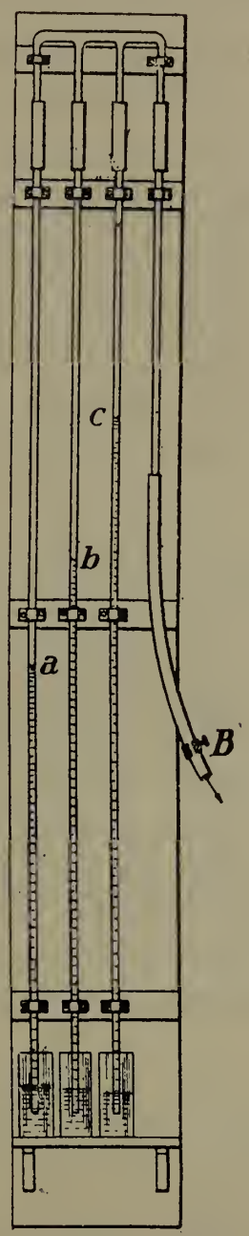


Fig. 28

the pressure of the remaining air. The pressure per unit area exerted by a column of liquid depends only upon the height of the column and the density of the liquid. If the heights to which the liquids rise in the tubes are represented by $a, b,$ and $c,$ (Fig. 28) and their respective specific gravities

by d_1, d_2 and d_3, the quantities d_1a, d_2b and d_3c, are proportional to the pressures per unit area at the bottom of each tube due to the liquids. But this pressure is the same for each tube and hence

$$d_1a = d_2b = d_3c \quad . \quad . \quad . \quad (1)$$

If one of the liquids, say the second, is water, its specific gravity $d_2=1$, and the specific gravity of the other liquids can be obtained from the relations $d_1=\frac{b}{a}$ and $d_3 = \frac{b}{c}$, where a, b, and c are the heights of the columns of liquids which can be determined by direct measurement.

DIRECTIONS.—With the beakers filled with the liquids whose specific gravity is to be determined, decrease the pressure in the tubes by means of suction till the lighter liquid has risen almost to the top of the tube containing it. Close the pinch cock B on the rubber tube so as to keep the liquids at constant heights in the tubes. By means of the measuring rod and meter stick determine the height of the liquid in each tube above the level of the liquid in its respective beaker. Designate the height of the lighter liquid, the water and the heavier liquid, by a, b and c, respectively, and let d_1, d_2 d_3 be the corresponding specific gravities. If d_2 is taken equal to unity, the specific gravity of the lighter liquid is found from the relation $d_1 = \frac{b}{a}$ and that of the heavier liquid from $d_3 = {}^{b}$.

Change the pressure slightly in the tubes so as to cause the heights of the liquid columns to be different from what they were before and again determine the specific gravities of the liquids from measurements of the heights of the columns. Repeat the above, varying slightly the heights of the liquids each time, till five determinations of the specific gravities of the liquids have been made. Record the mean of the five determinations for each liquid.

According to the directions on the use of the Beaume hydrometer for determinating the specific gravity of liquids given under experiment 18, note the readings of the hydrometer in the vessels near the sink containing the lighter and the heavier liquids. Use great care in rinsing and drying the hydrometer before placing it in either solution. Compare the readings of the hydrometer with the values obtained above.

Questions.—1. What objection is there to having the vertical tubes of unequal diameter? 2. Assuming b to be correct what percent error in d_3 will an error of 2 *mm.* in measurement of c produce, (a) in your first determination, (b) in your last determination?

20. SURFACE TENSION.

(a) To determine the value of the surface tension of water by capillary tubes. (b) To determine the value of the surface tension of water by means of the Jolly balance. (D. 207-211; G. 138; K. 254-257, 263; W. 156-159.)

Part 1. *Surface Tension by Capillary Tubes.*

Apparatus.—Capillary tube, stand and scale for holding tube, beaker of distilled water, mercury, fine balance.

Description and Theory.—The surface of a liquid is in a state of tension similar to the tension of a tightly stretched drum head. This tension is manifested by the tendency of the surface to contract so as to enclose as small a volume as possible, as is seen in a rain drop. Imagine a line drawn in any direction across the surface of a liquid.

The value of the force upon unit length of this line is called the "surface tension" of the liquid. By virtue of this force, a liquid which wets a tube will rise in it above the general surface of the liquid when the tube is placed vertically in

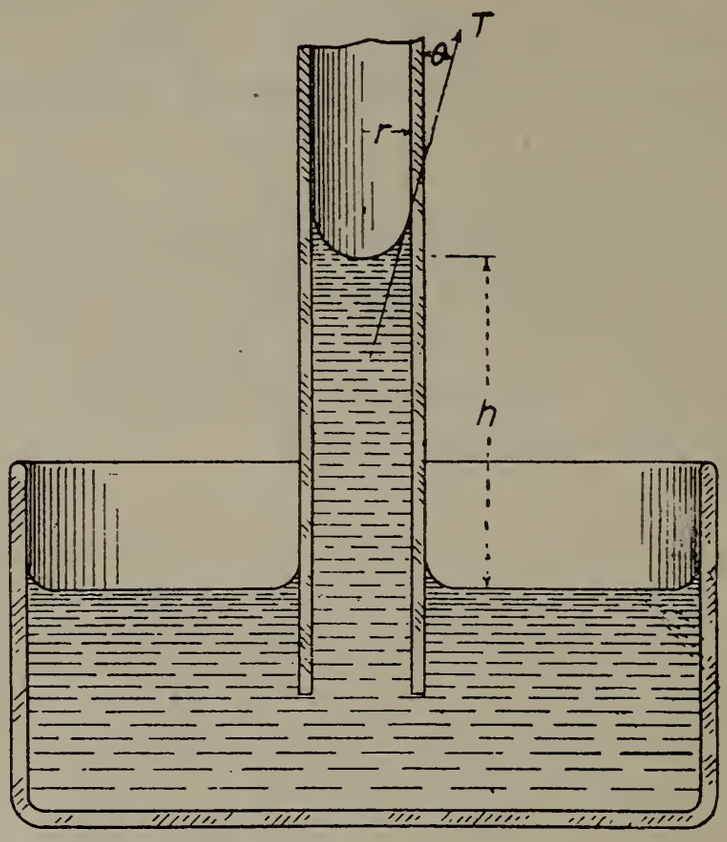


Fig. 29

the liquid. As soon as the liquid in the tube rises above the normal surface, gravity tends to pull it downward. The liquid will then rise in the tube only to such a height that the maximum tension which the surface can exert is just balanced by the weight of the liquid in the tube.

If T is the surface tension, or the force exerted by the surface on a line one centimeter long, and r is the radius of the tube (Fig. 29) the total force acting around the circumference of the tube is $2\pi r T$. If the surface of the liquid makes the angle θ with the walls of the tube at the point of contact, the total force acting upward along the wall of the tube is

$2 \pi r T \cos \theta$. The weight of the liquid which this force supports is $\pi r^2 h d g$, where r is the radius of the tube, d the density of the liquid, h the height to which the liquid rises and g the acceleration of gravity. Then,

$$2 \pi r T \cos \theta = \pi r^2 h d g \quad . \quad . \quad . \quad (1)$$

from which

$$T = \frac{r h d g}{2 \cos \theta} \quad (2)$$

In the case of very small tubes, the cosine of the angle of contact θ differs little from unity. Hence (2) may be written in the form

$$T = \tfrac{1}{2} r h d g \quad . \quad . \quad . \quad (3)$$

From this equation it is seen that, if the height h to which a liquid of density d rises in a tube of radius r is known, the value of the surface tension is readily obtained.

Directions.—Take a glass tube of small bore and clean it carefully with a caustic potash solution, then rinse it thoroughly in running water. After cleaning avoid putting the finger over the end of the tube as this might soil the inner surface. To measure h, fasten the tube to the vertical scale by means of rubber bands. Place the scale and tube vertically in a beaker of distilled water with the base upon which the scale is erected resting on the edge of the vessel. Lower the tube and raise it again to be sure that it is wet as high as the column will rise. Note the position of the surface of the liquid on the scale by sighting through the walls of the beaker along the under surface of the liquid. Read the position on the scale to which the water rises in the tube. From the readings made determine the height h of the water column in the tube. Move the tube up or

down a few millimeters and repeat the above measurements. Make five determinations of h and take the mean value.

To measure r the radius of the capillary tube, see that the tube is perfectly dry, weigh it on the fine balance, and then draw into it a thread of mercury about 3 or 4 *cm.* in length and weigh again. The difference between the two weights is the weight of the mercury in the tube. Measure the length of the mercury thread, l. The weight is

$$m = d \pi r^2 l \quad . \quad . \quad . \quad . \quad (4)$$

and

$$r = \sqrt{\frac{m}{13.55 \pi l}} \quad . \quad . \quad . \quad (5)$$

where the density of mercury, d for 20°C = 13.55.

Substitute in equation (3) the values obtained, and determine the value of the surface tension. Repeat the above determination with a tube of different diameter.

Slight impurities on the surface, such as an imperceptible quantity of oil from the fingers, may change the value of T very much. The vessel and tube should therefore be carefully cleansed and rinsed with distilled water. The fingers should not be allowed to touch the water.

QUESTIONS.—1. How does h vary with the diameter of the tube? 2. Would this method be applicable to liquids which do not wet the walls of the tube? Explain your answer.

Part 2. *Surface Tension by Means of the Jolly Balance*

APPARATUS.—Jolly balance, set of known masses, wire rectangle, beaker of distilled water.

DESCRIPTION AND THEORY.—If a wire in the form of a rectangle is immersed in a liquid and then raised, it is observed that the liquid clings to the wire in the form of a

thin film. Suppose a wire in the form of a rectangle is suspended from the spring of a Jolly balance and the position of the scale read when only the ends of the rectangular wire dip in the liquid. Then, if the rectangle is immersed in the water and afterwards the spring is moved up till the film which is formed just breaks and the position of the scale read, the difference between the readings in the two cases gives a measure of the pull due to the surface tension. If a represents the average elongation of the spring per gram and e is the elongation which represents the pull of the film on the wire, $\frac{e}{a}g$ is the force due to surface tension, g being the force on one gram due to gravity. If the rectangle has a width W, then

$$T = \frac{e}{a}\,\frac{g}{2W} \qquad . \quad . \quad . \qquad (1)$$

The factor 2 is introduced because the film has two surfaces.

Directions.—Place known masses 2, 3, and 4 grams on the pan of the balance and determine the elongation a of the spring per gram added. (See experiment 17.) Clean the wire rectangle by dipping it in a caustic potash solution and then holding it in running water. Handle it by means of the fine wire attached to the middle of the frame. Fasten the wire rectangle to the hook under the pan and lower it into a beaker of distilled water. Raise the spring slowly and note approximately the depth of the ends of the rectangle below the surface of the water when the film breaks. Repeat this two or three times. Now lower the rectangle till the ends of the wire project into the water to a depth equal to the depth found above and observe the reading on the scale. Call this the zero reading. Again lower the wire into the water and raise the spring *very slowly* till the film on the wire just breaks. Note the reading of the vernier. The

difference between this reading and the *zero* reading gives the elongation *e* of the spring due to the surface tension of the film. Measure the width *W*. (Fig. 30.) of the wire

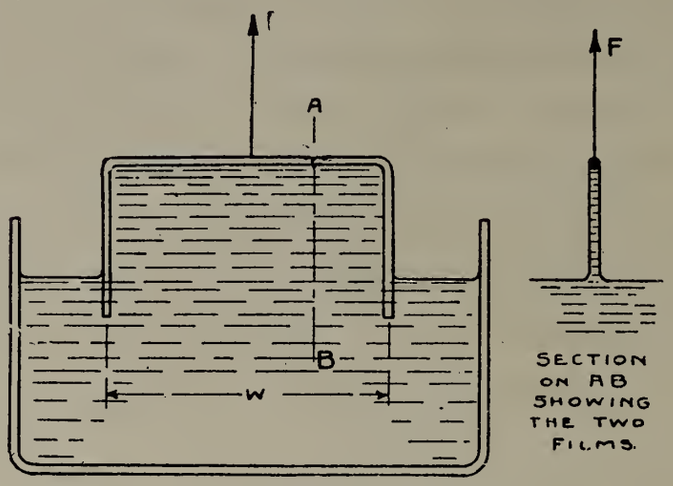


Fig. 30

rectangle and substitute the values determined in equation (1) and solve for *T*.

Repeat these determinations four more times and find the mean value of the surface tension found for the five trials.

QUESTIONS.—Is the surface tension in the film any greater just before it breaks than it is just after it begins to form? Explain your answer.

21. BOYLE'S LAW.

To show that at a constant temperature, the pressure of a given mass of gas varies inversely as its volume. (D. 219, 221, 294, 295; G. 168, 181,182; K. 198-201; W. 130, 133, 134.)

APPARATUS.—Boyle's law apparatus, barometer, thermometer, pan.

Theory, Method and Description.—For a given mass of gas the product of its pressure and its volume varies directly as its *absolute* temperature. This is frequently called the gas law. If p be the pressure, v the volume and T the absolute temperature of a gas, the general formula for the gas law may be written

$$pv = RT$$

where R is a constant for the given gas. Boyle's law is a special case of this gas law and holds only when the temperature remains constant, during a change of pressure or of volume, under which conditions pv equals the constant RT. It may be stated as follows:—*The temperature of a given mass of gas remaining constant, its pressure varies inversely as its volume.* The apparatus for obtaining the data to show that this law holds true in the case of air, consists of two vertical glass tubes joined at the bottom by a rubber hose, and adjustable in height on vertical supports. This apparatus is similar to figure 32, but the bulb B is replaced by a simple closed tube, the other tube being open. The tubes are partly filled with mercury, confining a quantity of air in the closed tube, and the height of the mercury columns may be read on a scale between the supports by means of the adjustable indicator and vernier. The mercury in the open tube is under atmospheric pressure only, but by varying the height of the two tubes the pressure on the confined air in the closed tube may be varied. By varying the pressure and volume of the air column and noting the heights of the mercury columns, and the corresponding volume of the air column, as well as the barometric pressure, we can show that the pressure times the volume remains constant, provided we take care to keep the temperature constant. Instead of finding the actual volume of air it is more convenient to

assume the cross section of the tube containing the air column to be constant, and to measure the length of this air column which is, of course, directly proportional to the volume. If p, v, and p', v', be corresponding values of pressure and volume for two different lengths l and l' of the air column of constant cross section a, then $p\,v = p'\,v'$ or $p\,a\,l = p'\,a\,l'$ from which

$$p\,l = p'\,l'$$

It is therefore only necessary under the given assumption to measure corresponding values of pressure and length of air columns to verify Boyle's law. When the mercury columns are of equal height, the pressure on the air columns is that of the atmosphere. When the height of mercury in the open tube is greater than that in the closed tube, the pressure on the confined air is greater than one atmosphere, and the weight of a column of mercury of unit cross section and having a height equal to the difference in the heights of the two mercury columns must be added to the atmospheric pressure in order to get the total pressure per unit area on the confined air. But since barometric pressures are expressed in cm. of mercury, we need only add the difference in height of the two mercury columns to the barometric height to express the pressure on the air enclosed in the tube. If the height of mercury is greater in the closed tube, the pressure on the air column is less than atmospheric pressure because the latter must also support a column of mercury of a height equal to the difference of the two mercury levels. The total pressure will be the difference between the barometric height and the difference in level of the two mercury columns.

The mercury in the cistern of the Fortin barometer must be adjusted by means of the screw beneath the instrument

until the surface just touches the ivory tip which represents the zero of the scale. The barometer height should be carefully read on the attached scale and vernier. There are certain corrections to be made in this reading. Since mercury in a capillary tube is depressed below its free level, the height of mercury in the barometer tube is less than it should be by an amount depending on the diameter of the tube. This correction should be added to the barometer reading. Since on the other hand the brass scale of the barometer, and the mercury expand with an increase of temperature, the density of the mercury will be less, and the reading will be greater at ordinary temperatures than it would be under standard conditions; that is at 0° Centigrade. We must therefore subtract a certain correction from the barometric height as read on the scale. A table of these corrections will be found in the appendix.

For very careful work a correction should also be made for variations in the acceleration of gravity at different positions on the earth's surface, but the correction may be neglected in the present case.

In this particular experiment we are dealing only with relative pressures, and as the barometer is at practically the same temperature as the other mercury columns and the expansion of the brass scale is small compared with that of the mercury, the temperature correction may be omitted. The two glass tubes are of the same bore, hence the capillary action is balanced in them and no capillary correction is needed in the heights of the two mercury columns.

Directions.—*Caution.*—Be careful not to lower the open tube so far that the mercury may run out. Also avoid handling the closed tube or in any way heating it, as small changes in temperature produce an appreciable change in

volume. When the volume of the air column has been changed by altering the level of the mercury, always give the air time to recover from the resulting change in temperature before reading the height of the mercury columns.

Attach the thermometer to the closed tube in order to see that the temperature of the enclosed air remains constant. Record the temperature to 0.1° and see that it is the same after each change in the height of the mercury before taking a reading. Adjust the height of the tubes until the mercury stands near the top of the open tube. Read to 0.01 cm. the height "*a*" of the column in the open tube and the height "*b*" of that in the closed tube. Read also the height "*c*" of the top of the enclosed air column. Lower the mercury in the open tube about 10 cm. and read the new heights *a* and *b*. Continue this process until the mercury in the open tube is near the bottom. Then raise the mercury again by stages of about 10 cm. each, reading the corresponding heights *a* and *b* at each stage. Read the barometer height and make the necessary corrections for capillarity. Calculate the pressure $p = h + (a-b)$ at each position ("*h*" being the corrected barometric height), and the corresponding length $c-b$. With this data show that Boyle's law holds true.

QUESTION.—a) Show the form of the graph for Boyle's law by plotting corresponding values of pressure and volume of your data on coordinate paper.

b) Plot a curve having products of p and v as ordinates and corresponding values of p as abscissae.

Experiments in Heat.

22. CALIBRATION OF THERMOMETERS.

(a) To determine the zero and steam points of a thermometer. (b) To calibrate the scale of a thermometer. (D. 267; G. 300-304; K. 365-368; W. 180.)

Apparatus.—A 100°C thermometer, a thermometer with an uncalibrated scale, hypsometer, copper calorimeter, ring stand, gas tubing, Bunsen burner, glass funnel, beaker.

Description and Theory.—A mercury thermometer consists of a thin glass bulb connected with a capillary tube. The bulb and part of the capillary tube are filled with mercury and then the tube is sealed off after all the air has been expelled. There are two definite temperatures that may be used to locate two fixed points on the thermometer. One is that of melting ice which, on the Centigrade thermometer, is marked as 0° and the other is the temperature of vapor formed over water boiling freely under a pressure of 76 cm. of mercury. This point is marked 100° on the Centigrade thermometer.

When an ordinary thermometer is used for any sort of accurate work it is generally found to be quite badly in error. Glass is very unstable in its molecular structure. As it gets older the molecules gradually readjust themselves and a sort of annealing process goes on similar to that occuring when glass is gradually cooled after being worked in a flame. This change in the walls of the thermometer alters the volume contents of the thermometer bulb and tube, so that the fixed points of the two definite temperatures will be found to have changed. In making standard thermo-

meters the glass must be cooled very slowly and allowed to become quite old before they are calibrated, in order that the annealing process may be fully completed. Because of the change in the boiling and freezing points of the thermometer, there is, of course, a corresponding change in the position of the other divisions of the thermometer.

When the space between the two fixed points is divided into equal divisions the bore of the capillary tube is supposed to be uniform throughout. This is not really true so that, for very accurate work it is necessary to calibrate and make a calibration curve for the thermometer.

The fixed points, the 0° and the 100° points, of a thermometer are tested by noting the readings indicated by the thermometer when it is placed in melting ice and in dry steam.

If t_o is the reading of the calibrated thermometer when the bulb is in melting ice then the correction at zero degrees is

$$C_o = [O^\circ - t_o] \quad . \quad . \quad . \quad . \quad (1)$$

Again if the temperature of steam at the given pressure is T_s and the reading of the thermometer when placed in the steam is t_s, the correction at the boiling point is

$$C_s = [T_s - t_s] \quad . \quad . \quad . \quad . \quad (2)$$

Assuming the bore of the calibrated thermometer to be uniform, which is allowable for ordinary work, the correction for any intermediate temperature t_t is given approximately by

$$C_t = [C_o + \left|\frac{C_s - C_o}{100}\right| t_t] \quad . \quad . \quad (3)$$

Therefore the correct temperature t_c, when the thermometer reads t_t, is given by

$$t_c = t_t + [C_o + \left|\frac{C_s - C_o}{100}\right| t_t] \quad . \quad (4)$$

The zero point and the steam point of the uncalibrated thermometer can be located in the same manner as the corresponding points of the calibrated thermometer. The intermediate points can be calibrated by placing the two thermometers in the same vessel of water at various temperatures and, having observed the readings of both, by designating the point on the uncalibrated thermometer by the corrected reading of the calibrated thermometer for each temperature of the water in which they are immersed.

The hypsometer, Fig. 31b, used in locating the steam

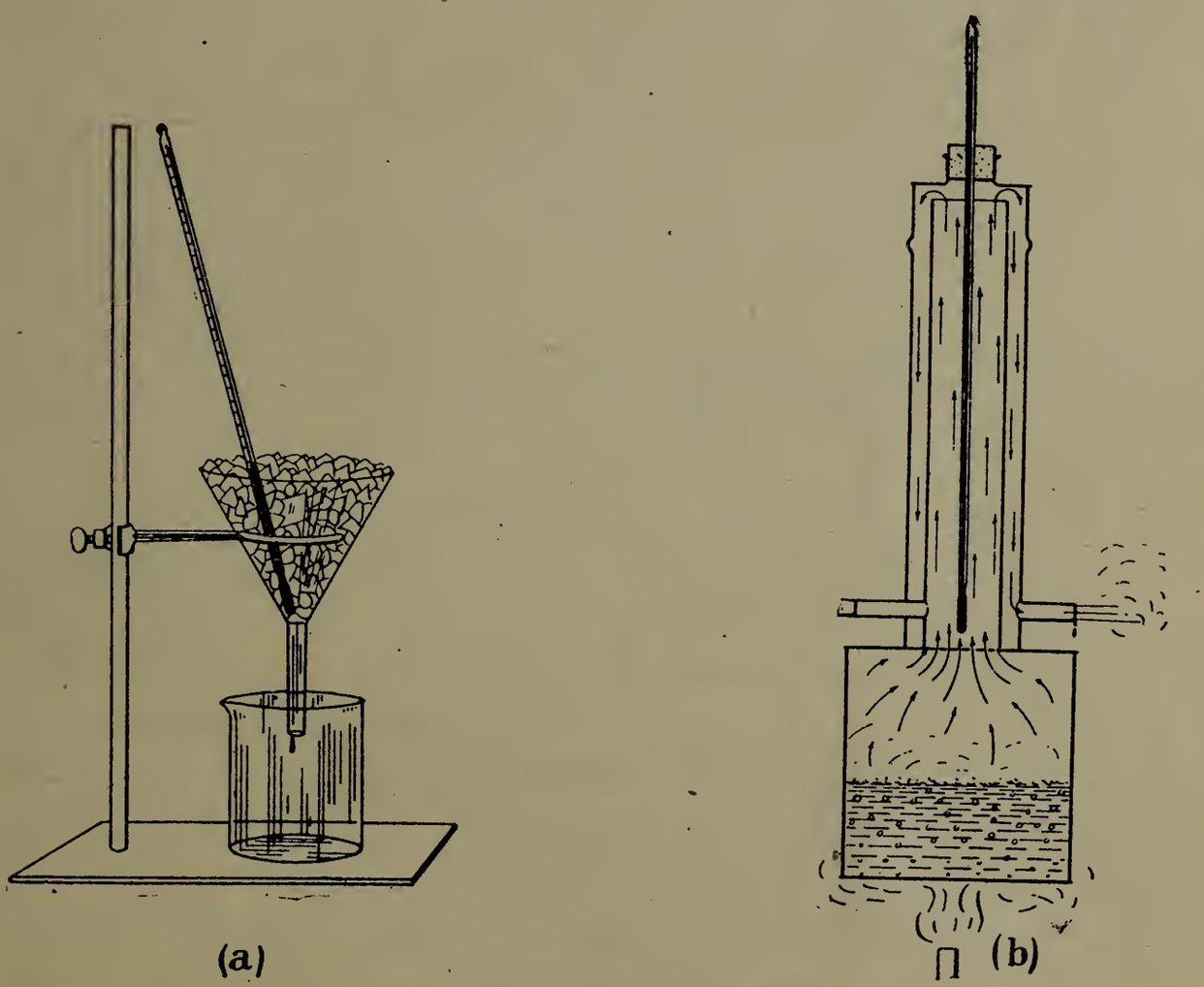


Fig. 31 a and b

point, is a copper vessel having a reservoir in the lower part for holding water and in the upper part having an inner and

an outer jacket. The two jackets are so constructed that the steam generated in the reservoir has to pass up through the inner jacket and down through the space between the two jackets before escaping through the outlet tube. If the thermometer is inserted in a cork in the top of the hypsometer it is thus surrounded by dry steam.

DIRECTIONS.—The freezing point should be tested first. To do this, arrange a glass funnel in a ring support as shown in Fig. 31a, with a beaker beneath to catch the melted ice. Fill the funnel with snow or cracked ice and insert the thermometer so that the entire thread of mercury is beneath the surface of the melting mixture. After the thermometer has been in the mixture long enough for its reading to reach the melting temperature, take readings t_o, every minute until three readings have been obtained. Read carefully to tenths of a degree. At the time of each reading raise the thermometer so that the top of the thread of mercury may be seen just above the surface of the cracked ice and then quickly take an accurate reading.

In order to test the boiling point, pour water into the reservoir of the hypsometer till it is about two-thirds full and place a Bunsen flame under it. This may be done while the freezing point is being tested. *Do not let the hypsometer boil dry.* Insert the thermometer in the hypsometer as shown in Fig. 31b, so that the 100° point may be seen just above the top of the cork. After the steam has been escaping freely for some time from the tube leading from the outer jacket, and the mercury in the thermometer has reached a steady point, take readings t_s every minute until three readings have been obtained. Read carefully to tenths of a degree.

Read the barometer and, having corrected the reading

according to the table of corrections in the Appendix, find the boiling temperature T_s, corresponding to that corrected pressure, by referring to the steam table in the Appendix.

It is found that after the thermometer has been exposed to the steam the glass bulb does not contract to its former volume for some time afterwards. So that, if the thermometer is placed again in the melting ice (after being allowed to cool 30° or 40° below the boiling point) the position of its freezing point will have changed slightly.

After the thermometer has acquired the temperature of the freezing mixture take readings every minute until three readings have been obtained. In the calibration of the uncalibrated thermometer the average of these readings will be taken as the freezing point t_o of the calibrated thermometer instead of that obtained before the thermometer was heated. Substitute in equations (1) and (2) and find the corrections C_o and C_s.

In a similar manner note the readings indicated by the uncalibrated thermometer when placed first in dry steam and then in melting ice. Draw a scale in your note book similar to the one upon which the thermometer is mounted and locate upon the scale the position of the temperature of steam and the zero point just observed.

In order to fix intermediate points on the uncalibrated thermometer, place the two thermometers in a calorimeter containing water at about 20°C. Observe the position of the mercury column when they have become stationary, and having corrected the reading t_t of the calibrated thermometer according to equation (4), record this corrected reading on the scale in your note book at the position indicated by the mercury column on the uncalibrated thermometer scale. In a similar manner locate and determine the value of the position on the uncalibrated scale when the ther-

mometers are immersed in water at temperatures of about 40°, 60° and 80°C.

QUESTIONS.—What is the difference between the averages of the two sets of readings obtained for the freezing point before and after the boiling point readings? Explain the cause of this difference.

23. CHARLES' LAW.

PRESSURE COEFFICIENT OF AIR.

To show that the volume of a given mass of gas remaining constant, the pressure varies directly as its *absolute* temperature. (D. 264, 265, 279, 293, 294; G. 332, 333; K. 376, 393-397; W. 195, 197.)

APPARATUS.—Air thermometer, hypsometer, tripod, Bunsen burner, copper can, stirrer, mercury thermometer, and cloth.

THEORY, METHOD AND DESCRIPTION.—(See theory under experiment 21.) Charles' law (also called Gay Lussac's law) is a special case of the gas law, and holds only when the pressure of a gas remains constant while its volume and temperature vary. Under these conditions the ratio of volume to absolute temperature is

$$v/T = R/p \text{ (a constant).}$$

The law may therefore be stated as follows:—*The pressure of a given mass of gas remaining constant, its volume is directly proportional to its absolute temperature.* It is quite evident that, when the volume of the gas is kept constant,

its pressure is directly proportional to its absolute temperature, or

$$p/T = R/v \text{ (a constant)}.$$

This may be looked upon as another way of expressing Charles' law, and it is this law which we wish to verify. The gas constant R may be shown to be equal to $\alpha\, p_o v_o$ where p_o and v_o are the pressure and volume respectively of the given mass of gas at zero degrees centigrade and α is *the pressure coefficient, or that fractional part of the pressure at zero degrees Centigrade by which it increases for each degree Centigrade rise in temperature.* That is to say, the pressure of the given mass of gas at t degrees Centigrade will be

$$p_t = p_o\,(1 + \alpha t)$$

and

$$\alpha = (p_t - p_o)/(p_o\, t)$$

or the pressure coefficient will be the change in pressure per unit pressure at zero degrees Centigrade per degree Centigrade change in temperature.

From the relation $p_t = p_o\,(1 + \alpha t)$ we see that at the temperature

$$t = -1/\alpha,$$

p_t will equal zero. *This point in the scale of temperatures' at which there is no energy in the gas, is called the absolute zero,* and temperatures measured from this point are called absolute temperatures. On the Centigrade scale the absolute zero is—273°, and

$$T^\circ \text{ (abs,)} = (t + 273)^\circ\text{C}.$$

The apparatus used to verify this law is similar to the Boyle's law apparatus, but the closed tube leads into a horizontal capillary tube a foot or more in length which ends in

a vertical elongated bulb, *B*. This arrangement makes it possible to place the bulb in baths at different temperatures, and also keeps these baths at some distance from the mercury columns. See Fig. 32. The volume of air can be kept constant by adjusting the mercury each time after the air has come to the temperature of the bath, so that it stands at a mark on the apparatus just at the beginning of the capillary tube. The temperature of the air in this tube will be

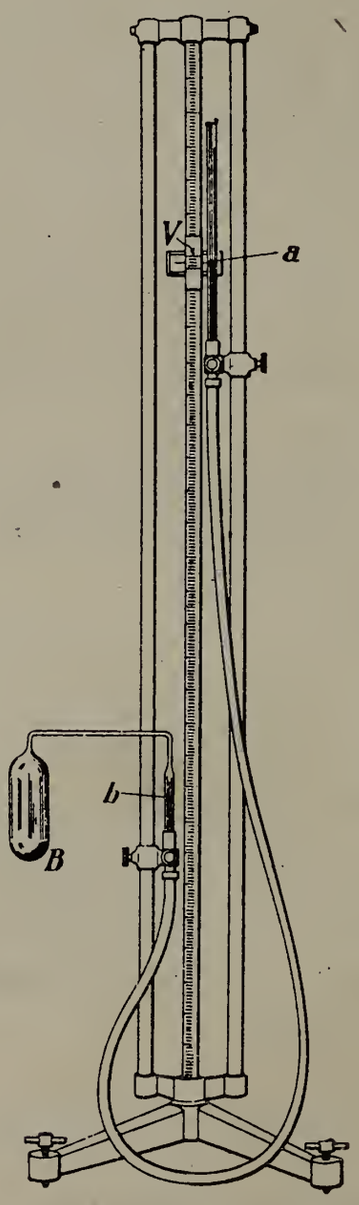


Fig. 32

slightly different from that of the bath, but its volume is so small in comparison with the volume of air in the bulb,

that we may consider all of the air to be at the same temperature. The volume of the bulb does not remain absolutely constant but increases as the glass expands with a rise in temperature, and as the pressure of the confined air increases.

By observing the temperatures of the baths, and the corresponding pressures due to the atmospheric pressure on the open tube and the difference in level of the mercury columns, we obtain the data for verifying the law and calculating the pressure coefficient.

The law holds only approximately for gases which do not obey Boyle's law, but for air the change in the ratio p/T is very slight and the pressures may be used to find the corresponding temperatures over a wide range by means of a calibration curve. The apparatus is therefore often called an air thermometer.

Directions.—Adjust the mercury columns until the height of that in the open tube is at least 15 cm. less than that in the closed tube. Place the copper can (without stirrer) on a stand so that the bulb projects into it, being careful not to break the latter nor the capillary tube. Fill the can with snow or finely crushed ice until the bulb is covered. Fill the interstices of the ice with cold water. Allow several minutes for the air in the bulb to come to the temperature of the ice bath, *i. e.* 0° C. Adjust the tubes until the mercury is at some well defined mark near the beginning of the capillary tube, being careful that it does not run into the capillary tube and the bulb. Note the height of the mercury column "*a*" in the open tube and the height "*b*" in the closed tube and record the difference in level $(a-b)$. Remove the ice from the can and replace it in the ice box at the sink. Fill the can with water and heat it to about 25 degrees C. Then remove the burner and allow the water and the air in the bulb to come to a common tem-

perature, using the stirrer to equalize the temperature of the water. Note this temperature. Adjust the height of the mercury so that it stands at the same mark on the closed tube as before, in order to make the volume of air the same, and observe the new difference in level of the mercury columns. Then heat the water to about 60 degrees and proceed to find the new difference in level at this temperature. Then carefully lower the level of the mercury column until its height in the open tube is again the first value of "*a*", before removing the hot bath, so that when the air in the bulb cools and contracts, the mercury will not run over into the bulb. Next fill the can of the hypsometer about one-half full of water, place it on the tripod so that the bulb of thermometer projects into the hypsometer, and adjust a cloth around the neck of the bulb to prevent steam from escaping freely. Then heat the hypsometer until steam is generated freely and again adjust the mercury columns until the volume of air is the same as it was before. Observe the difference in level of the mercury columns and also the corrected barometric pressure *h*. (See experiment 21 for corrections to barometric readings.) Before lowering the temperature of the enclosed air by removing the steam bath, be sure to lower the level of the mercury to the original value "*a*", so that no mercury flows into the bulb when the air contracts.

Obtain the temperature of the steam at the barometric pressure from the steam table in the appendix, and calculate the total pressure corresponding to each temperature used.

With the data taken show that Charles' law holds true, and calculate the pressure coefficient of air for constant volume, and the absolute zero.

Comparison of Temperatures by the Air and the Mercury Thermometers.—With the data taken for melting ice and for steam, plot a straight line curve having total pressure of

enclosed air as ordinates and corresponding temperatures as abscissae, assuming the pressure coefficient to be a constant throughout this range of temperatures. On this curve find the temperatures corresponding to the pressures observed when the bulb was immersed in the baths at about 25 and 60 degrees C. respectively. Compare each of these temperatures with the corresponding reading on the mercury thermometer. Find also the absolute zero on the Centigrade scale by prolonging the curve until it meets the axis of temperatures.

Question.—What are the reasons for and against the use of the air thermometer as compared with the mercury in glass thermometer?

24. LINEAR EXPANSION.

To determine the coefficient of linear expansion of a metal rod. (D. 273-275; G. 314-321; K. 379-388; W. 184-186.)

Apparatus.—Linear expansion apparatus and metal rod, steam generator, Bunsen burner, tripod, thermometer, micrometer screw, funnel and waste water jar.

Theory, Method and Description.—*The coefficient of linear expansion of a substance is the change in length of unit length of the substance for a change in temperature of one degree Centigrade.* For instance if a metal rod changes from a length L_1 cm. to L_2 cm. when the temperature changes from t_1 to t_2, the total change in length per cm. of its original length is $(L_2-L_1)\ /L_1$ and the change per degree change in temperature will be

$$\alpha = \frac{L_2-L_1}{L_1(t_2-t_1)}$$

For isotropic bodies this coefficient is the same in all directions. Strictly speaking it is not a constant but depends

upon the temperature, and in order to define it precisely we should refer to the change in length of unit length of the substance at zero degrees Centigrade. For all practical purposes however, (since the change with temperature is very small) α is calculated by the formula given above.

The method used is to place a metal rod about one meter long in a bath of cold water, noting the temperature of the water and measuring the length of the rod when it has come to the same temperature; then to replace the water bath by one of steam, again noting the temperature and measuring the length of the rod after allowing sufficient time for it to reach the temperature of the steam. With this data the coefficient of linear expansion may be calculated by the formula given.

The metal rod is surrounded by a sheet copper jacket, its ends projecting through rubber corks which fit tightly about the rod and within the ends of the jacket. See Fig. 33.

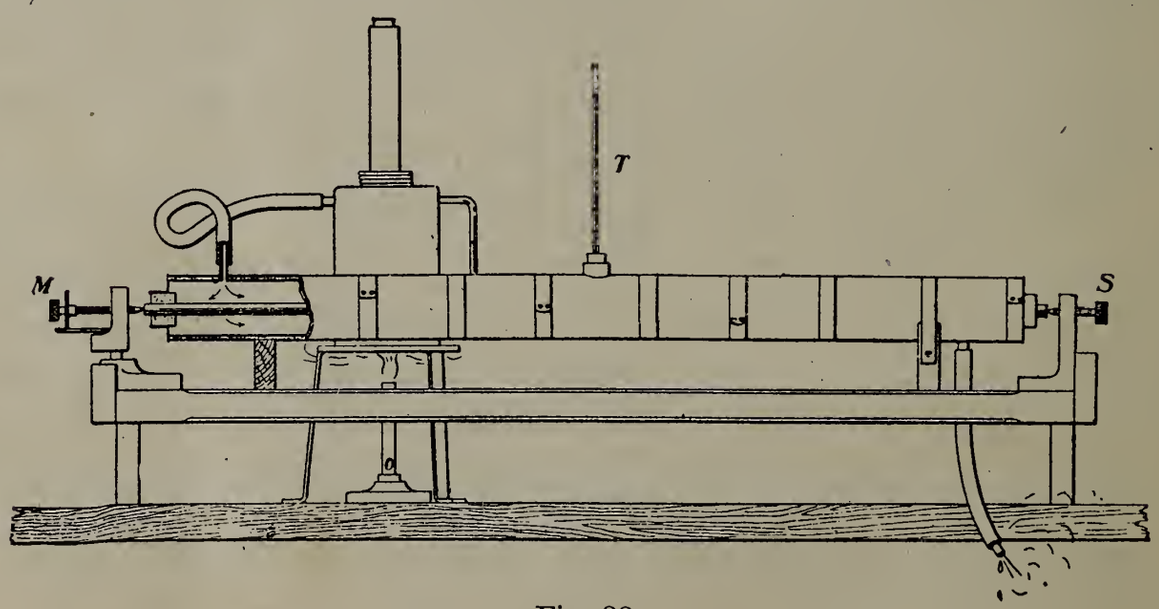


Fig. 33

The jacket is covered with sheet asbestos to prevent loss of heat by radiation, etc., and rests upon a wooden support. At one end of this wooden support is an adjustable screw *S* which may be turned into contact with the rod. At the

other end is a micrometer screw *M* which can also be turned into contact with the rod and serves to measure its elongation. To measure the temperature of the bath and rod a thermometer *T* is passed through a cork in an opening at the middle of the upper side of the jacket. A second opening at one end serves to introduce the water and steam and a third opening at the other end of the under side is fitted with a rubber tube for carrying off the water or steam and may be closed by means of a pinch clamp.

Directions.—To save time the water may be heated to the boiling point while the rest of the experiment is being performed. Fill the steam generator about one-half full of water, and heat it with a Bunsen burner, placing it far enough from the rod to prevent heating the latter. Meantime close the lower outlet with the pinch clamp and fill the jacket with cold water, being careful not to cause an overflow. Adjust the thermometer in the upper opening so that it rests on the rod and allow sufficient time for the rod and the water to come to a common temperature before noting the latter. Adjust the brass set screw so that it touches the end of the rod, then adjust the micrometer screw until it is about five millimeters from the end of the rod and the zero of the graduated circular disc is at the edge of the straight mm. scale. Then turn the screw into contact with the rod, counting the number of turns and noting the number of divisions on the last fraction of a turn. (Since there are 100 divisions on the circular scale 9 complete turns and 45 divisions over would give 945 divisions, etc.) In order to tell when the screw and rod come in contact an electric buzzer is used. (See experiment 13.) Turn the micrometer back to the starting point but leave the set screw as it is. Measure the length of the bar in cm. with the beam compass. Allow the water to run out into the sink and

leave the lower outlet open. When steam is being freely generated, connect the generator to the upper end-opening by means of rubber tubing and allow it to flow through the jacket, noting the change of temperature on the thermometer. When steam issues freely from the lower outlet and the temperature remains constant at about 98°, see that the set screw is in contact with the rod. If it is not in contact, do not move the screw but press the rod gently into contact. Then turn the micrometer screw until it just touches the end of the rod, noting the number of divisions through which you turn it. Then record the temperature of the bar in the steam. The pitch of the micrometer screw is 1/2 mm. hence there are 2000 of the divisions on the circular scale in a cm. Find the expansion of the rod, the difference of the micrometer readings in cold water and in steam. From the data obtained calculate the coefficient of expansion of the rod.

Repeat the experiment, and calculate the mean coefficient from the two determinations.

QUESTION.—a) How would your result be affected if the rod were not in contact with the screw *S* after expansion.

b) What force in pounds weight would be needed to stretch the rod the same amount as in this experiment?

25. COEFFICIENT OF CUBICAL EXPANSION OF A LIQUID.

To determine the coefficient of cubical expansion of a liquid. (D. 277; G. 327, 328; K. 389; W. 189-191.)

APPARATUS.—Specific gravity bottle, liquid, thermometer, fine balance, set of known masses, calorimeter, tripod, Bunsen burner.

Description and Theory.—The specific gravity bottle (Fig. 34.) is simply a small bottle which is provided with an accurately fitting ground glass stopper. A very small hole through the center of this stopper leads to the interior of the bottle, its object being to allow the bottle to be completely filled with any liquid.

The method used in this experiment to find the coefficient of cubical expansion of a liquid, depends upon the relative expansion of the liquid and glass. Let m_o be the mass of the

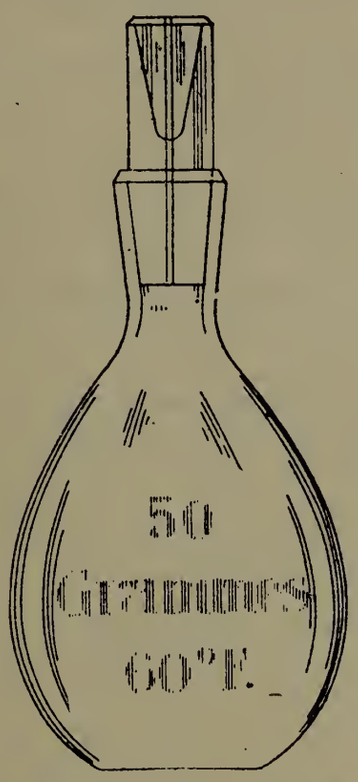


Fig. 34

bottle and liquid at O°C, d_o the density of the liquid at the same temperature and m the mass of the bottle. Then the volume of the liquid at O°C will be

$$v_o = \frac{m_o - m}{d_o} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If β is the coefficient of cubical expansion of the liquid, its volume at t°C will be

$$v = v_o(1+\beta t) = \frac{m_o - m}{d_o}(1+\beta t) \quad . \quad (2)$$

Similarly, the capacity of the bottle at O°C is

$$v_o = \frac{m_o - m}{d^\circ} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and, if the coefficient of cubical expansion of glass is α, its capacity at t°C will be

$$v' = v_o(1+\alpha t) = \frac{m_o - m}{d_o}(1+\alpha t) \quad . \quad (3)$$

The volume, at t°C, of the liquid expelled is therefore

$$\frac{m_o - m}{d_o}(1+\beta t) - \frac{m_o - m}{d_o}(1+\alpha t) = \frac{m_o - m}{d_o} t\,(\beta - \alpha). \quad (4)$$

This volume is also given by $\frac{m_o - m_1}{d_o}(1+\beta t)$ where m_1 is the mass of the bottle and liquid at the temperature t. Therefore,

$$\frac{m_o - m_1}{d_o}(1+\beta t) = \frac{m_o - m}{d_o} t\,(\beta - \alpha). \quad . \quad . \quad (5)$$

Solving (5) for β, we find

$$\beta = \frac{m_o - m_1}{(m_1 - m)t} + \frac{m_o - m}{m_1 - m}\alpha \quad . \quad . \quad (6)$$

Since $m_o - m$ and $m_1 - m$ differ very little, equation (6) may be written.

$$\beta = \frac{m_o - m_1}{(m_1 - m)t} + \alpha \quad . \quad . \quad . \quad (7)$$

This equation expresses the coefficient of cubical expansion of the liquid in terms of the coefficient of cubical expansion of glass, the original mass of liquid in the bottle at zero degrees, the final temperature t, and the mass of the liquid that is expelled from the bottle when its temperature is changed from O° to t°C.

Directions.—Clean and dry the specific gravity bottle very carefully, then find its mass m on the fine balance. Fill the bottle with the liquid whose coefficient of expansion is to be determined, place it in the calorimeter and pack snow or shaved ice around it. See that none of the ice gets into the bottle. Place the stopper also in the ice, and after leaving the bottle and stopper in the ice for 10 minutes, dry the stopper quickly, but thoroughly with a cloth and put it in the bottle. See that no air bubbles cling to the stopper. Leave a small drop of the liquid on the top of the stopper so as to have the capillary tube filled in case there is a further contraction of the volume of the liquid in the bottle. Leave the bottle in the ice at least 5 minutes longer, then wipe off the top of the stopper with a cloth. Remove the bottle, dry it carefully, and then find the mass m_0 of the bottle and contents on the fine balance. The weighing should be done as quickly as possible, as a precaution against the loss of weight through evaporation of the liquid that may escape as its temperature begins to rise.

Heat some water (not closer than to within 10 or 12 degrees of the boiling point of the liquid whose coefficient is being determined,) and place the bottle in it. Do not let the water extend above the neck of the bottle. After leaving the bottle in the water 10 or 15 minutes, note the temperature t of the water, wipe off the liquid from the stopper, remove the bottle, and having dried it thoroughly, find its mass m_1 on the balance. Taking the value of α the coefficient of cubical expansion of glass, as 0,000025, substitute the values found for m, m_0, m_1 and t, in equation (7) and calculate the value of β.

In a similar manner make a second determination of the value of β.

Questions.—If a bubble of air is trapped in the bottle

when making the weighing m_o, what effect will it have on the value of β found? Explain your answer.

CALORIMETRY.

(D. 280-284; G. 452-458; K. 398-401; W. 199-201.)

In each of the following four experiments a quantity of heat is to be measured. The calorimeters in which this measurement is to be made, are metal vessels, usually covered with insulating material to prevent the loss of heat by radiation, and supplied with a stirrer for equalizing the temperature of the water which they contain.

The method usually employed is known as "the method of mixtures," which is based upon the principle of equal heat exchanges. That is, when two or more bodies, originally at different temperatures, are placed in thermal contact, and the heat exchanges take place exclusively between these bodies, the quantity of heat lost by one part of the system of bodies is equal to the heat gained by the other part. In this method a given mass M grams of the substance whose heat is to be determined is heated to a temperature t_2°C. and is quickly plunged into, or mixed with W grams of water at a lower temperature t_1°C. The mixture will then come to a uniform temperature t°C.

It is evident that besides the gain in heat by the water in rising to the temperature of the mixture, the calorimeter containing the water and any other bodies such as the thermometer and stirrer, which have a common temperature with the water will also gain in heat. The simplest way of taking these bodies into account is to treat them as so much extra water beyond the amount W; that is, we must find their water equivalent. *The water equivalent of a body is the amount of water which requires as much heat to raise*

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its temperature 1°C. *as does the body in question. It is equal to the mass of the body multiplied by its specific heat.*

In order to avoid errors due to gain or loss of heat by radiation from or to neighboring bodies, the following method of compensation is generally used. On starting, the water and calorimeter are brought to a certain temperature below that of the room and the process of heating is continued until they are at a temperature as much above that of the room, as they were originally below it. In this way approximately as much heat is gained by the calorimeter from the surrounding air until it comes to the same temperature as the latter, as is lost by it to the air, while it exceeds this temperature.

26. SPECIFIC HEAT.

To determine the specific heat of copper by the method of mixtures. (D. 280-284; G. 452-454, 457-466; K. 398-406; W. 199-207.)

Apparatus.—Calorimeter, steam generator, steam heater, two thermometers, copper or aluminum pellets, drying cup, tripod, wooden paddle, paper funnel, cloth, coarse balance and set of weights.

Theory, Method and Description.—Every substance requires a definite amount of heat to raise a given mass of the substance through a given temperature. *The quantity of heat necessary to raise the temperature of one gram of the substance through one degree C., at any given temperature, is called the specific heat of the substance at that temperature.* For scientific purposes the quantity is measured in terms of a unit called the calorie. *The calorie may be defined as the quantity of heat required to raise the temperature of one gram of water from* 15° *to* 16° C. (This is the

mean heat capacity of water between its freezing and boiling points). The specific heat of water at 15° C. is therefore unity.

In engineering practice quantity of heat is measured in British thermal units. *The B. T. U. is the heat required to raise the temperature of one pound of water, one degree Fahrenheit.*

The specific heat of copper or of aluminum is to be determined by the method of mixtures. Let W be the known mass of water in the calorimeter, at a temperature t_1, M the mass of the substance whose specific heat C is to be determined, t_2 its temperature, t the temperature of the mixture, and w the water equivalent of the calorimeter.

Then taking the specific heat of water as unity, the heat gained will be

$$(W+w)\ (t-t_1) \text{ calories.}$$

In falling to the temperature of the mixture, the heated body will lose an amount of heat

$$C\ M\ (t_2-t) \text{ calories.}$$

Equating the amounts of heat lost and gained, we have

$$C\ M\ (t_2-t) = (W+w)\ (t-t_1)$$

from which to calculate the specific heat.

The water equivalent of the thermometer is small and may be neglected, and if the calorimeter cup and stirrer are of the same material as the pellets, we may assume that they have the same specific heat. If their joint mass is m, their water equivalent will be Cm and the equation becomes

$$C\ M\ (t_2-t) = (W+Cm)\ (t-t_1)$$

from which we calculate C

To prevent heat becoming lost by radiation the copper calorimeter cup is surrounded by a hair-felt jacket enclosed within a second copper vessel. The calorimeter has a wood-

en lid which holds a copper stirrer for equalizing the temperature of the water. A thermometer passes through a rubber stopper in the middle of this lid and through the stirrer. The metal pellets are heated in a cup which is surrounded by a steam jacket.

Directions.—Fill the steam generator about half full of water and heat it with a Bunsen burner. Take enough metal pellets to fill the inner calorimeter cup about one-third full, and pour them into the small cup of the steam heater. The pellets should be perfectly dry. If they are not, pour them into the brass cup, and heat them over the flame, stirring meanwhile with the wooden paddle until they are dry. When they have cooled somewhat pour them into the small steam heater cup and insert the cork and thermometer. Work the bulb of the thermometer into the pellets, being careful not to break it. Connect the lower opening of the steam heater to the steam generator and allow steam to come out through the opposite upper hole, so that it passes all around the small can. Close the outer lower opening with a rubber hose and pinch clamp. After the steam begins to issue shake the heater occasionally and invert it, so that the pellets are thoroughly mixed and the temperature uniform throughout, being careful to hold the cork and thermometer in place. After a time the temperature will become stationary at 97° or 98°C. In the meantime weigh the inner calorimeter cup (dry) and record the mass in grams. Fill the cup a little more than half full of ice water and weigh again. Place cork and thermometer in place and adjust the length of the thermometer so that it will reach within about two centimeters of the bottom of the cup. Place the cup in the calorimeter and see that there is no ice in the water. Shake the steam heater once more and read the temperature to one-tenth degree. Then remove the cork and with the

help of the paper funnel pour the pellets into the ice water with one swoop. Do not delay to collect stray pellets but immediately insert the cork and thermometer, being careful not to break the latter by forcing it into the pellets. Shake the calorimeter for a few seconds, being careful not to spill any of the water, and watch the thermometer. Just as it ceases rising and begins to drop read the temperature to one tenth degree. Weigh the inner calorimeter cup again, with water, and pellets, included. From the data taken calculate the specific heat of the metal pellets.

Before repeating the determination note whether the temperature of the mixture was as much above that of the room as the original temperature was below it, and if not, determine by an inspection of the data whether more cold water is needed or more hot pellets, to obtain this temperature. With the altered amount of water or pellets, repeat the experiment and calculate again the specific heat of metal.

QUESTIONS.—*a*) What is the effect on your result, of spilling some of the water from the calorimeter while shaking it?

b) Which metals heat up the quicker, those with high or those with low specific heat?

27. THE MECHANICAL EQUIVALENT OF HEAT.

To determine the mechanical equivalent of heat by means of Callendar's apparatus. (D. 280-282, 290; G. 452-454, 505, 506; K. 409-411; W. 199-201, 249-251, 494.)

APPARATUS.—Callendar's apparatus with two 2000 gram weights, coarse balance and set of weights, beaker, funnel, bent thermometer, straight thermometer, six 50 gram weights and hose and nozzle for draining.

Theory, Method and Description.—The first law of thermodynamics may be stated as follows:—*Whenever mechanical energy is converted into heat, or heat into mechanical energy, the ratio of the mechanical energy to the heat is constant.* This constant ratio is called *the mechanical equivalent of heat. It is the number of units of work which must be done in order to produce unit quantity of heat energy.* To determine this constant we shall use the following method:—A cylindrical brass calorimeter containing a known quantity "W" of water at a known temperature "t_1" is rotated against the friction of silk bands which bear upon it owing to known weights which are suspended from them. On starting, the water and calorimeter are at a certain temperature below that of the room, and the calorimeter is turned until it is at a temperature as much above that of the room as it was below at the start. The total number of revolutions "n" of the calorimeter and the new temperature "t_2" of the water are noted and the circumference "c" of the calorimeter measured. Since the calorimeter also absorbs heat in rising to this temperature we must calculate its water equivalent "w" or the amount of water which would require just as much heat as the calorimeter does to be raised through unit temperature. If the average force at the circumference of the calorimeter is the weight of a mass m, the work done in overcoming this resistance through a distance equal to n times the circumference c is $n\ c\ m\ g$. The quantity of heat required to raise the water and calorimeter from the temperature t_1 to the temperature t_2 will be $(W+w)\ (t_2-t_1)$. The ratio of these two quantities will be the mechanical equivalent of heat "J," or

$$J\ (W+w)\ (t_2-t_1) = n\ c\ m\ g.$$

The apparatus (See Fig. 35) consists of the mounted cylindrical brass calorimeter C rotated by hand by means of

a crank wheel, the number of revolutions being registered by a counter on one side. Unequal weights M_1, M_2 are suspended from the ends of a silk belt which bears on the cylin-

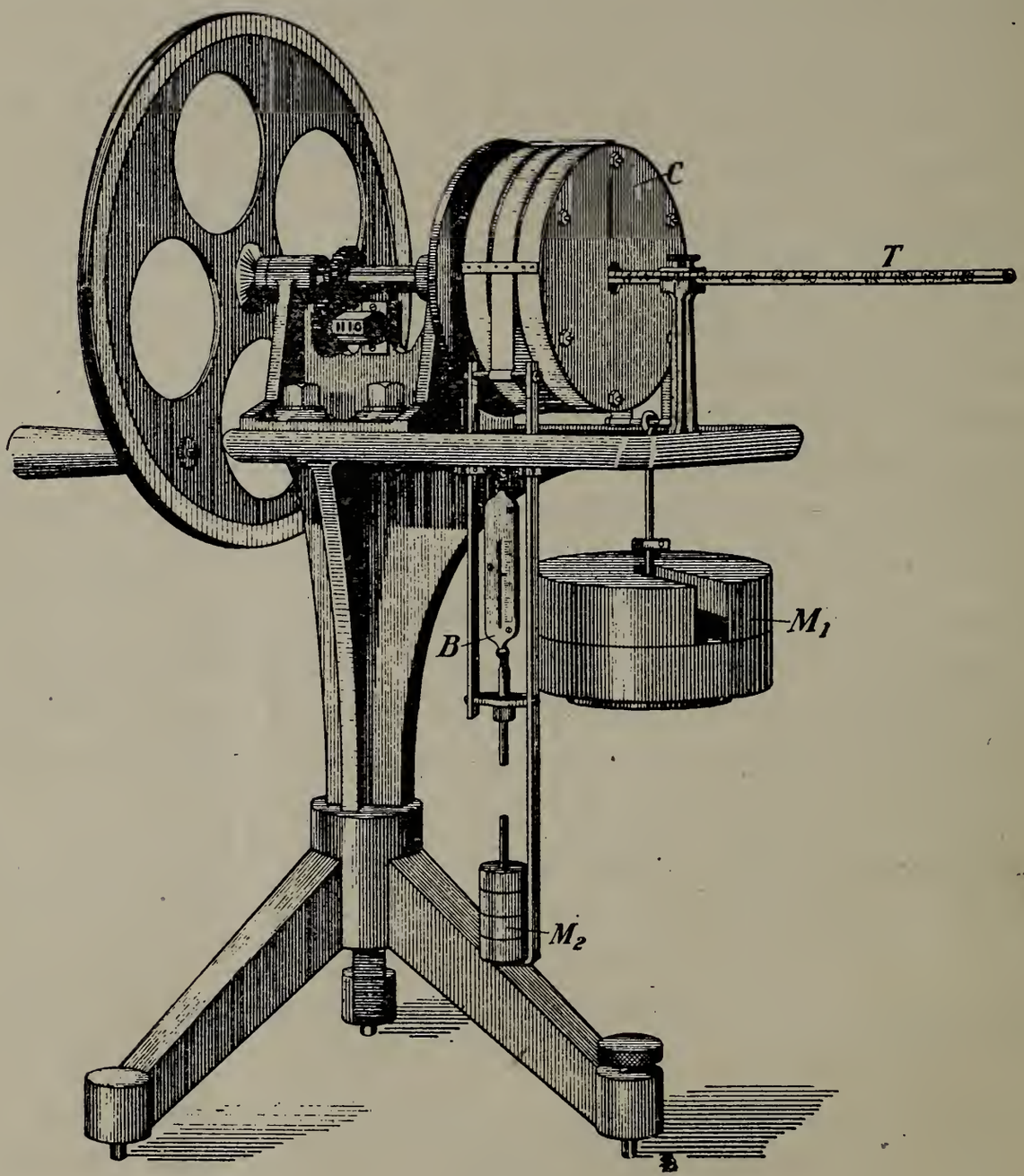


Fig. 35

der and makes one and a half complete turns around it. A light spring balance B is arranged so that it acts in direct opposition to the lighter weight. The motion of the belt is limited by means of stops, and the weights are adjusted and the cylinder rotated so that the weights are held suspended,

the final adjustment being made automatically by the spring balance. The temperature of the water is read on a bent thermometer T inserted through a central opening on the front end of the cylinder.

Directions.—Hang the 4000 grams from the end of the belt opposite the spring balance and add small brass weights to the other end until "floating equilibrium" is

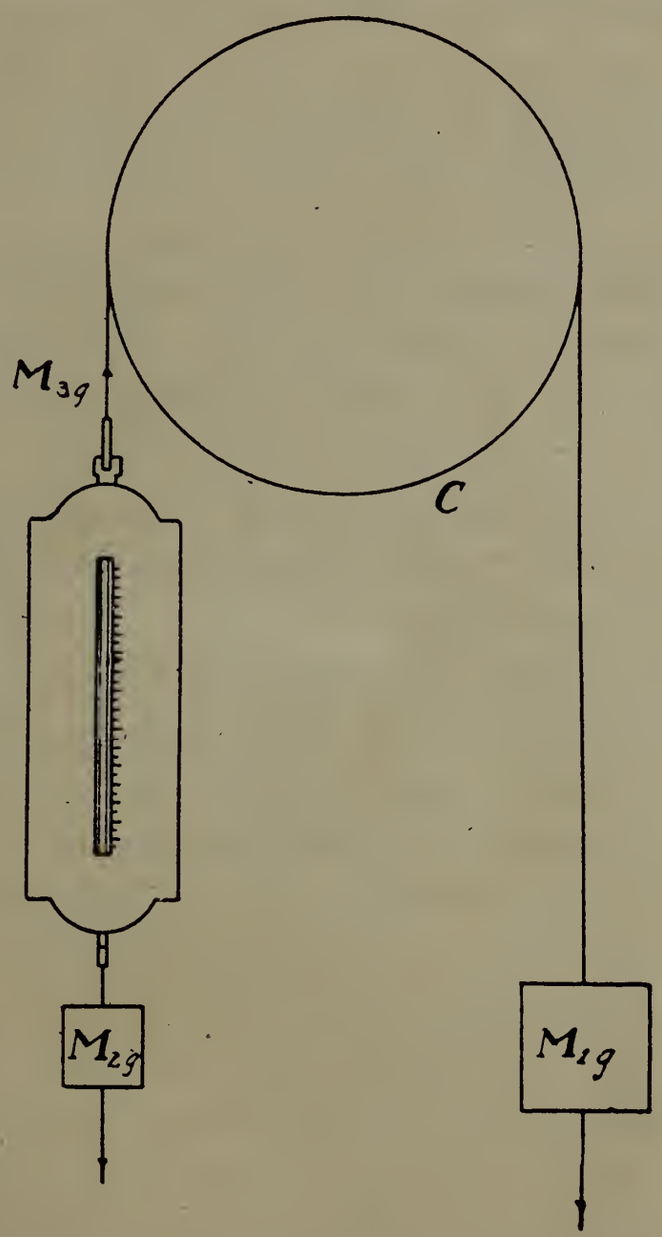


Fig. 36

maintained when the cylinder rotates; that is until the weights are not supported at the stops, but hang freely from the belt. Level the apparatus so that no part of this

floating system touches the frame. The student chosen to rotate the cylinder should start it a number of times to become familiar with its behavior. Hang the straight thermometer near the calorimeter, allow it to come to room temperature and note this temperature. Place about 200 grams of distilled water in a beaker and cool it to about 7° C. below the room temperature. Weigh funnel, beaker and water, then pour the water into the calorimeter through the central opening, and reweigh funnel and beaker with whatever water may be adhering to them. Pass the bent thermometer through the central opening (being careful not to put any strain on it by pressing against the walls of the cylinder,)and fasten it so that its bulb is immersed in the water. Remove the weights from the belt and turn the cylinder until the thermometer indicates a steady temperature, after which replace the weights. Then rotate the cylinder with a steady motion. A second student should note the number of revolutions on the counter and at every 45th and 95th revolution he should call "ready", and at every 50th and 100th revolution he should call "read". A third student should note the temperature of the water at the word "read" and a fourth student should record all data. The cylinder should be rotated until the temperature of the water is about 7° C. above the temperature of the room. The water should then be drawn from the calorimeter by removing the flat headed brass screw and replacing it by the nozzle and hose. Calculate the length of the circumference of the cylinder by winding a string several times around it, measuring its length, and finding the length of a single turn. The average force exerted at the circumference c will be the weight of the 4000 grams $M_1 g$ plus the spring balance reading $M_3 g$ minus the small brass weights $M_2 g$. (See Fig. 36.)

With the data obtained calculate the mechanical equivalent of heat, assuming $g = 980$ cm/sec.^{2}.

Question.—If it requires 4.2 joules to raise one gram of water 1°C. in temperature, what is the value of "J" a). in foot pounds per calorie? b). In foot pounds per B.T.U? c) In foot poundals per B. T. U.?

28. LATENT HEAT OF FUSION.

To determine the latent heat of fusion of ice. (D. 280-283, 307, G. 452-458, 467, K. 398-401, 424-426, W. 199-201, 208-211.)

Apparatus.—Calorimeter with lid and stirrer, hot water reservoir with lid and stirrer, water heater, thermometer, ring stand, tripod, Bunsen burner, coarse balance, set of weights and drying cloths.

Theory, Method and Description.—If a solid be heated slowly, when its melting point is reached it will be found that the temperature remains constant at this point until all the substance is changed into the liquid state, all of the heat supplied being used up in this change of state. Each substance requires a definite amount of heat to transform a given amount of the substance from the solid to the liquid state at the temperature of the melting point. *The latent heat of fusion of the substance is the number of calories of heat required to change one gram of the substance at the melting point from the solid to the liquid state.*

To determine the latent heat of fusion of ice we may use the method of mixtures (see Calorimetry.) A mass M grams of ice at 0° C. is added to a mass W grams of water at t_2°C. in a copper calorimeter. The temperature t of the water must be noted at the instant when all of the ice is melted. The specific heat of copper may be taken as

0.093 calories per gram per degree Centigrade, and if the mass of the calorimeter and stirrer is m grams, their water equivalent w will be

$$w = 0.093\ m \text{ grams}$$

Since the specific heat of water is unity the quantity of heat gained by the ice in melting and rising to the temperature of the mixture after it is melted, will be $M\ L_f + M(t - O°)$ calories, where L_f is the latent heat of fusion of the ice. The heat lost by the water and calorimeter in dropping to the temperature t will be $(W+w)\ (t_2 - t)$ calories. Equating these two quantities we have

$$M\ (L_f + t) = (W+w)\ (t_2 - t)$$

from which we calculate L_f.

To avoid error in the result, the ice should be carefully dried before placing it in the calorimeter. To avoid error due to radiation, the final temperature of the water should be as much below, as its original temperature was above that of the room.

The calorimeter consists of a copper cup with a wooden lid and copper stirrer, surrounded by a hair felt jacket enclosed within a second copper vessel.

The hot water reservoir is a copper vessel with lid and stirrer, covered with a hair felt jacket and having at the bottom an outlet for the water. The lids and stirrers of the calorimeter and reservoir are pierced with holes to accommodate the thermometers.

DIRECTIONS.—Fill the steam heater about three fourths full of hot water and heat to about 60°. Run this hot water into the reservoir, being careful to have the lower opening closed with the rubber tubing and pinch clamp. Weigh out about 60 to 80 grams of ice in lumps about the size of walnuts, and place them on a strip of cloth. Note the temperature of the room. Dry the inner calorimeter cup and stirrer

with the hot air blast and weigh them. Then run hot water into the cup and add cold water until the temperature is about fifteen degrees above that of the room. Have the water fill the cup about four-fifths full. While one student weighs the calorimeter with water and stirrer, as quickly as possible, the other should dry the lumps of ice with the strip of cloth. After stirring a few seconds to equalize the temperature of the water, read the thermometer to 0.1° and quickly insert the lumps of ice *under* the stirrer. Stir the water and note the change of temperature as the ice melts. If the ice is all melted before the temperature is as much below, as it originally was above that of the room, or if there are still lumps of ice present when this temperature is reached, treat this as a preliminary trial from which to determine the proper amount of ice to use. With this amount of ice proceed as before and when the ice is just melted read the thermometer quickly. Weigh the calorimeter with the stirrer, water and melted ice and from the data taken calculate the latent heat of fusion of ice. Repeat the experiment, using more or less ice, or colder or hotter water to fulfill the given conditions if necessary.

Question.—How would your results be affected a) if the ice were not all melted when the temperature of the mixture was taken? b) If an appreciable amount of the ice had melted between weighing it and placing it in the calorimeter? Give reasons for your answers.

29. LATENT HEAT OF VAPORIZATION.

To determine the latent heat of vaporization of water. (D. 280-283, 308, 314; G. 452-454, 457, 458, 468; K. 398-401, 432-442; W. 199-201, 213, 214.)

Apparatus.—Calorimeter with stirrer and glass tube,

water trap, connecting hose, steam boiler, thermometer, barometer, tripod, Bunsen burner, brass cup, coarse balance, set of weights, and cloth.

THEORY, METHOD AND DESCRIPTION.—If a liquid be heated slowly, when its boiling point is reached it will be found that the temperature remains constant at this point until all the liquid has been changed to a vapor, all of the heat supplied being used up in this change of state. Each substance requires a definite amount of heat to transform a given amount of the substance from the liquid to the vapor state at the temperature of the boiling point. *The latent heat of vaporization of the substance is the number of calories of heat required to change one gram of the substance at the boiling point from the liquid to the vapor state.*

To determine the latent heat of vaporization of water, we may use the method of mixtures. Steam is passed into W grams of water at a temperature t_1 degrees (below that of the room) in a calorimeter until the temperature t of the water is as much above, as it originally was below that of the room. The mass M of the condensed steam is obtained from the increase in mass of the water. The calorimeter consists of a copper cup surrounded by a hair felt jacket and enclosed within a second copper vessel. It has a wooden lid and a copper stirrer for equalizing the temperature of the water which it contains. If the mass of the calorimeter cup and stirrer is m and their specific heat 0.093 calories per gram per degree Centigrade, their water equivalent w will be

$$w = 0.093\ m \text{ grams.}$$

The temperature t_2 of the steam may be obtained from the barometric pressure at the time of the experiment by referring to the steam table in the Appendix. Since the specific heat of water is unity, the heat gained by the calorimeter and water will be $(W + w)\ (t - t_1)$ calories,

and the heat lost by the steam in condensing and by the resulting water in dropping to the temperature of the mixture will be $M\ L_v + M\ (t_2 - t)$ calories, where L_v is the latent heat of vaporization. Equating these two quantities we get

$$M\ (L_v + t_2 - t) = (W + w)\ (t - t_1)\ .$$

from which we calculate L_v.

The temperatures t and t_1 are chosen equidistant above and below the room temperature respectively in order to avoid errors due to radiation.

Any drops of water which may be carried into the calorimeter by the steam will of course produce an error in the result, for the heat of vaporization of these drops has not been available for raising the temperature of the water in the calorimeter. To avoid this error the steam is passed through a water-trap before entering the calorimeter. This trap consists of a glass tube about 2.5 cm, in diameter and 10 cm. in length, closed at the ends by corks. The steam enters and leaves the trap by glass tubes which pass eccentrically through the corks. They are placed out of line so that the steam can not pass directly from one into the other with its full velocity, carrying moisture with it. The outlet tube extends nearly to the top of the trap, the free end passing vertically through the lid of the calorimeter to within about one centimeter of the bottom. Any moisture carried over by the steam or condensing in the tube will collect in the lower part of the trap.

Directions.—Fill the steam generator about half full of water and heat it. Dry the inner calorimeter cup and stirrer and weigh them on the coarse balance. Note the temperature of the room and the barometric pressure, on the thermometer and barometer in the laboratory. Fill the calorimeter about four-fifths full of water and cool it with

lumps of ice to about 10° below the room temperature. Then remove any ice which may be left in the calorimeter, insert the stirrer, and carefully weigh the cup with water and stirrer. When steam issues freely from the generator, connect the latter to the steam trap by means of the rubber tube, and allow the steam to flow for a few minutes. Note the temperature of the water and calorimeter, remove any moisture that may have collected in the trap, by taking out one of the corks, replace the cork and quickly insert the tube through the lid and stirrer. Stir the water while the steam is condensing and note the rise in temperature. When the water is about 10° above room temperature, quickly remove the tube, stir the water and note the temperature when it reaches its maximum. Then weigh the cup with stirrer, water, and condensed steam. From the data taken calculate the latent heat of vaporization of water.

Repeat the experiment four or five times, varying the amounts of the different factors as may be found necessary, and using freshly cooled water each time.

QUESTION.—How would your results be affected a) if an appreciable amount of solder (specific heat = 0.04) forms part of the weight of the calorimeter cup? b) If the steam were introduced into the calorimeter at a depth of 15 cm. under the surface of the water? Give reasons for your answers.

30. HYGROMETRY.—HUMIDITY AND DEW POINT.

To determine the dew point and relative humidity of the air in the laboratory. (D. 309; G. 437-442; K. 447, 448; W. 220, 221;

APPARATUS.—Nickel plated cup, copper stirrer, vessel

for holding ice water, two thermometers, cloth, wet and dry bulb hygrometer (mounted in room.)

Description and Theory.—The atmosphere consists of air mixed with water vapor. Ordinarily it is not saturated though saturation may be brought about either by an increase in the amount (mass per unit volume) of water vapor without change of temperature, or from a lowering of the temperature without change in the amount of vapor present. An hygrometer is an instrument designed for the determination of the amount of water vapor in the air. Two of the most common types are the dew point hygrometer and the wet and dry bulb hygrometer. The *dew point* is the temperature to which the atmosphere must be cooled in order that it may be saturated by the amount of water vapor actually present. In its simplest form the dew point hygrometer consists of a metal vessel whose outer surface is polished. By adding a cold mixture to the vessel, its temperature may be reduced until moisture from the atmosphere is deposited on the polished surface. The temperature, t_d, at which this occurs is the dew point. Knowing this and the temperature of the air, the absolute and relative humidities may be obtained from hygrometry tables. *The absolute humidity is the number of grams of water contained in one cubic meter of air. The relative humidity is the ratio of the mass of water vapor actually present in a cubic meter of air to the mass which a cubic meter of air would contain if saturated at the temperature.*

The wet and dry bulb hygrometer consists of two sensitive thermometers mounted on the same stand, the bulb of one being covered with cloth which is kept continually moist by being connected with a vessel of water. Unless the atmosphere is saturated with moisture there will be constant

evaporation from the cloth covering the bulb and this will cause the wet bulb thermometer to indicate a lower temperature than the other. The drier the air the more rapidly will evaporation take place and the greater will be the difference between the readings of the two thermometers. With these readings the various hygrometric values such as dew point, relative humidity, etc., may be obtained from tables constructed from the results of experiment.

DIRECTIONS.—Note the temperature of the laboratory near the place where the experiment is to be performed. Make a mixture of crushed ice and water in the cup for ice water. Clean and polish the nickel cup, pour water from the faucet into it to a depth of about a centimeter and then add ice water slowly until dew forms on the polished surface. In the meantime, stir the mixture with the stirrer. Do not breathe on the polished surface as this will cause a film of moisture to appear. In case the depth of the water in the cup gets to be more than about three centimeters without dew forming, pour out part of the water and continue to add ice water as before. When dew finally forms it will cover the polished surface to the height that the water stands on the inside. Note the temperature of the water at the *first* appearance of the dew. Now let the water warm up or, if need be, add faucet water in small portions to the mixture, noting the temperature when the dew begins to disappear. The mean of these two readings will give the temperature of the dew point.

Repeat the above determination two more times, drying and polishing the can each time. Take the average of the three values for the temperature of the dew point.

In case the dew point is below O°C, the mixture will have to be cooled by the addition of salt and shaved ice.

Refer to the hygrometry table in the appendix, and determine the saturation humidity at the temperature of the dew point. This is evidently the *absolute humidity* of the air in the room. Calculate the relative humidity.

Now take the thermometer readings on one of the wet and dry bulb hygrometers in the room (the one nearest the place where you have determined the dew point) after first being sure that the cloth is wet. In order to get the proper depression the wet bulb thermometer should be fanned or some other device should be employed for maintaining a rapid motion of the air past the covered bulb. If this is not done the air in the immediate neighborhood of the thermometers will soon become more moist than the bulk of the air in the room and evaporation will take place less rapidly than it should.

Record the reading of the wet bulb thermometer as t_w, that of the dry bulb thermometer as t, and the reading of the barometer as b. From the hygrometry table find p_w, the maximum vapor pressure for water vapor at the temperature t_w. The actual pressure p exerted by the water vapor present in the atmosphere (at temperature t) depends upon the density of the air into which evaporation takes place, that is, upon the barometric height b, as well as upon the velocity of the air currents. The empirical formula

$$p = p_w - 0.00075\, b\, (t - t_w)$$

gives the pressure approximately when the air surrounding the thermometers is in moderate motion. In the above formula p, p_w and b are expressed in cm. of mercury and $(t - t_w)$ in degrees centigrade.

Having found p, the dew point t_d (the temperature at which this pressure produces saturation) may be obtained

from the hygrometry table. Compute the relative humidity of the air in the room from the data obtained.

QUESTIONS.—(1) Explain why drops form on the outside of a pitcher containing cold water. Does the saying that this "indicates an approaching rainstorm" have any scientific foundation? Explain your answer. (2) What will be the effect on the final result for the relative humidity if the breath of the observer strikes the wet bulb thermometer when the data is being taken?

Experiments in Magnetism and Electricity.

31. MAGNETIC FIELDS OF FORCE.

To study the field of force about magnets and about conductors carrying a current of electricity. (D. 367, 368, 426, 427; G. 699, 700, 833, 836; K. 486, 487, 674, 676, 679; W. 418-420, 472, 476, 513, 514.)

Apparatus.—U shaped magnet, bar magnet, six pieces of cardboard, five paper outlines of size and shape of magnets, six sheets of blue print paper in a closed tube, six clips, pins, iron filings in a sifter, rectangular and circular coils of wire, solenoid, reversing key and small compass, connecting leads with plug and lamp resistance.

Theory and Method.—A magnetic field of force is the space surrounding a magnet or a conductor carrying a current of electricity, throughout which it exerts a magnetic force. We may think of this space as threaded with lines of magnetic force. The latter may be defined as lines drawn in the field, such that at every point of a line, the direction of the resultant magnetic force at that point is tangent to the line. The direction of such a line is the direction in which a free north-seeking magnetic pole would move. Hence the lines pass from the N pole of a magnet to the S pole through the surrounding medium. When iron filings are brought into this field, each small piece of iron becomes a temporary magnet by induction, and places itself tangent to a line of force with its north-seeking pole pointing in the direction of the line. The filings may be sprinkled upon sensitive paper in the field and exposed to sunlight. If the filings are then removed and the paper developed, the image formed

by the shadow of the filings will show the shape and density of the lines of force.

The lines of force about a straight conductor carrying a current of electricity are concentric circles. If the conductor be grasped in the *right* hand with the thumb pointing in the direction of the current, the fingers will encircle the conductor in the direction of the lines of force. If the conductor is wound in the form of a flat coil the lines of force about it are no longer concentric circles, but a large number of them will encircle all of the loops. If the conductor is wound in a solenoid (a long cylindrical coil with an axis much greater than its diameter) the field of force is still more distorted and, on the outside, resembles closely that of a bar magnet, the lines passing from one face (or end) to the other face of the coil and back to the first face through the coil.

DIRECTIONS.—*Caution.*—Do not expose the blue print paper to the light any longer than is necessary but keep it in the closed tube, taking out one sheet at a time. To get the proper time of exposure for this paper, cut a strip about 1/4 inch wide from one of the sheets and expose a piece an inch or two long to bright light (sunlight if possible) until it assumes a bronzed appearance. Then wash for a few minutes in clean water. If the print is a dark blue it has been exposed sufficiently long; if light blue the exposure has been too short; if grey blue it has been too long. In either of the last two cases make a second or third trial and note carefully the time of exposure required to obtain a dark blue print.

Fasten a sheet of blue print paper flat upon a cardboard by means of pins at the corners and place it symmetrically over the bar magnet in a place shaded from the sunlight.

(See Fig. 37). Sprinkle iron filings over the paper from the sifter, holding the latter about a foot above the paper in order to distribute them more uniformly. At the same time tap the cardboard gently with a pencil so that the filings arrange themselves along the lines of force, until the latter are distinctly outlined. If the filings are sprinkled on too thickly the lines will not be distinct. Next pin to the board

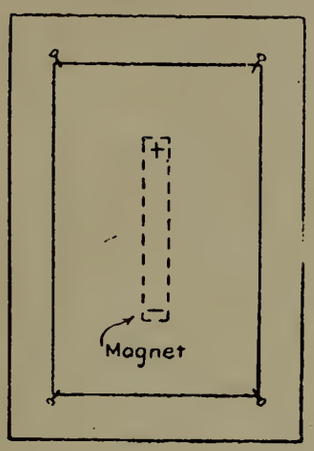


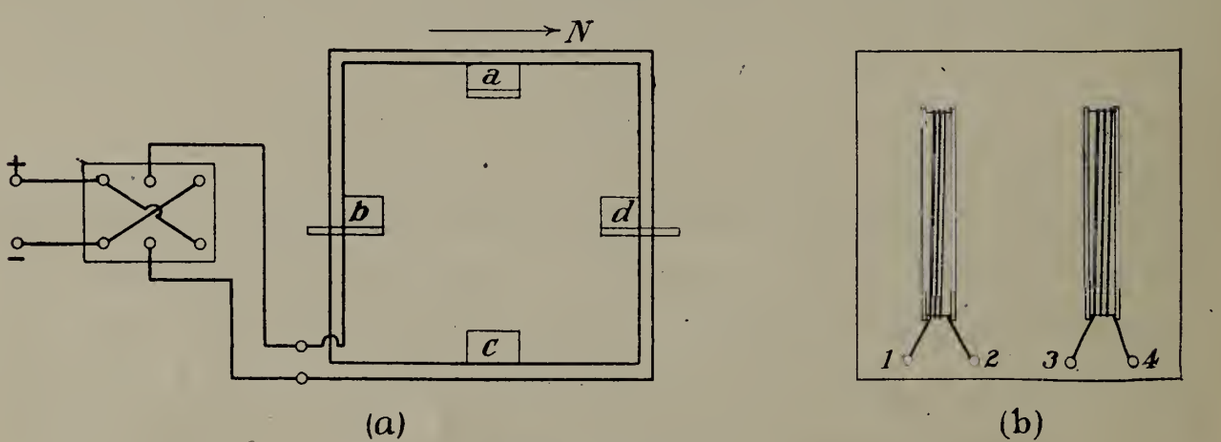
Fig. 37

just over the magnet a piece of thin card cut to the same size as the magnet, with a + and a − cut out at the ends, the + end coinciding with the N pole or marked end of the magnet and the − end with the S pole. Lift up the cardboard in a *vertical* direction from the magnet and place it carefully in the bright light at the window, exposing it for the proper length of time to get a dark blue print. Then pour the filings carefully into the pan provided at the sink, remove the paper from the cardboard, write your name upon the back in lead pencil, and place it face down in the water for about ten minutes or until the unchanged sensitizer has been washed out. In the meantime repeat the performance, using the bar magnet placed about 4 cm. in front of the U shaped magnet with like poles opposite each other. Then reverse the bar magnet and repeat again. Finally make a duplicate

set of these three prints so that each student has a set to bind in with his written report of the experiment.

When the prints are thoroughly washed out, hang them up to dry. Before binding them in your report mark arrow heads on ten or twelve distinct lines of force on each print, showing their direction.

Join the loose ends of the lamp cord to the terminals at one end of the reversing switch and place the porcelain plug in the Chapman receptacle in (or by) the bench. Always join conductors to terminals marked with the same sign (+ or −). The current will flow from the + to the − terminal. The reversing switch has its end terminals cross connected in such a way that when the source of current is connected to the middle terminals and the apparatus through



(a) (b)

Fig. 38a and b

which the current is to be sent is connected to the two terminals at either end, or vice versa, the current through the apparatus will be in one direction when the switch is closed through one end, and reversed when the switch is closed through the other end terminals. Join the middle terminals of the switch to the rectangular coil as shown in the diagram, Fig. 38a. A lamp joined in this circuit serves to cut down the current which would otherwise flow through the coil and prevents short circuiting when the loose terminals are acci-

dently brought into contact. Set the rectangular coil with its plane in the north and south meridian, and close the switch. Place the small compass on the shelf at the north side of the coil and move it around the conductor, noting on the data sheet the directions of the lines of force on each side by means of small arrows. The dots at b, d, etc., on the data sheet represent cross sections of the wire conductors. Do the same for the conductor at the south end of the coil. Then place the compass above and below the horizontal portions of the coil at *a* and at *c* and note the direction of the lines of force. Next reverse the direction of the current in the coil, by closing the switch in the opposite direction, and repeat the observation.

Replace the rectangular coil by the two circular coils (Fig. 38b.) and connect the coils so that the current of electricity flows through both in the same direction. With the aid of the compass determine the direction of the lines of force at the sides where the coils pierce the supporting table, and note the direction of the current in the coils at these points (whether up or down). This direction may be determined from the + and − signs on the connecting leads. Change the connections so that the current is reversed in *one* of the coils and proceed as before to note the relative directions of current and lines of force indicating these on the data sheet by means of small arrows.

Replace the coils by the solenoid and repeat the performance, recording the directions of the lines of force about the horizontal cross section represented on the data sheet. Compare this field with that of the bar magnet.

Question.—1.—What are some of the properties of lines of force, illustrated in these blue prints?

2.—Draw the magnetic field of force about a bar magnet and a compass needle, placed as shown in Fig. 42, and ex-

plain the effect produced upon the needle, by means of the properties of lines of force.

3.—Draw the field of force in a horizontal plane through the middle of a vertical coil of wire carrying a current of electricity and having its plane in the magnetic meridian with a compass needle suspended at its center, (tangent galvanometer, see Fig. 43) and explain the effect produced upon the needle, with the aid of the properties of lines of force.

32. ACTION BETWEEN MAGNETIC POLES

SUPERPOSITION OF MAGNETIC FIELDS

To study the law of action between magnetic poles and to show the effect of superimposing one magnetic field upon another. (D. 372, 380; G. 716-719, 736; K. 479, 485, 489-491; W. 416, 421, 427.)

APPARATUS.—Two bar magnets, small compass, stand with stirrup for suspending magnets, sheet of paper.

THEORY AND METHOD.—The law of action between magnetic poles is that "*like poles repel each other; unlike poles attract each other.*" This law is readily verified by suspending one of the bar magnets so that it is free to rotate in a horizontal plane about an axis perpendicular to its own, and approaching first one and then the other pole of a second magnet to the same pole of the suspended magnet. A magnet suspended as described forms a torsion pendulum and it performs harmonic vibrations. The characteristic of such motions is that the acceleration at any instant is proportional to the displacement from the position of rest, and the period of the vibrations is

$$T = 2\pi\sqrt{-\phi / \alpha}$$

where "ϕ" is the angular displacement and "α" the angular acceleration.

When a magnet of length "l" and pole strenght "m" is suspended in the magnetic field of strength H, making an angle ϕ with the direction of the field, the force acting on each pole is Hm, and the moment arm is $(l/2)\ sin\ \phi$ so that taking both poles into consideration, the moment of the couple tending to turn the magnet is

$$2mH\ (l/2)\ sin\phi = MH\ sin\ \phi$$

for the magnetic moment of the magnet, M, is equal to ml. But if I be the moment of inertia of the magnet, the turning moment acting on it will be $I\alpha$, and since the displacement and acceleration are always in opposite directions ϕ and α will always have opposite signs. Therefore

$$M\ H\ sin\ \phi\ =\ -\ I\ \alpha$$

and

$$-sin\phi/\alpha\ =\ I/\ (M\ H)$$

For small angles of vibration $sin\phi$ may be considered equal to ϕ itself, so that

$$-\ \phi\ /\ \alpha\ =\ I/\ (M\ H).$$

Substituting this value in the expression for the period, we have

$$T\ =\ 2\ \pi\ \sqrt{\frac{I}{HM}}$$

From this we see that the period of a vibrating magnet is inversely proportional to the square root of H, the strength of the field in which it vibrates, and of M, its magnetic moment.

When a second magnet is brought into the neighborhood of the suspended one, its field is superimposed upon that of the earth, and the strength of field acting on the latter magnet is decreased or increased according as like poles or unlike poles are placed adjacent to each other. In other words, since the predominating action is that between adjacent poles, the action of the second magnet upon the first is, in the second case the same as that of the earth, in the first case opposed to that of the earth.

If both magnets be suspended together, they may be considered as forming a single magnet, its magnetic moment being greater or less than that of either alone.

If the two magnets have their unlike poles in juxtaposition, and have nearly the same pole strength, the directive force tending to keep them in the magnetic meridian is very much reduced. Such a pair of magnets of very nearly the same strength, suspended with opposite poles adjacent is called an *astatic system* of magnets, and is very sensitive to influences tending to turn it from the magnetic meridian.

Since the earth is a large magnet having a magnetic north pole in the neighborhood of its geographic south pole and vice versa, any magnet placed in the earth's field sends lines of force to the earth's south magnetic pole and receives lines of force from the earth's north magnetic pole.

Unless a magnet be perfectly symmetrical about its axis and be in the magnetic meridian with its north seeking pole towards the north, the field of force surrounding it will be distorted. Any mass of iron in the earth's field and especially vertical iron pipes and columns, being magnetic by induction, distort the earth's field in their neighborhood. The distorted fields due to magnets may be studied by placing a small compass in the field and moving it continuously in the direction in which it points, marking the path followed

with a pencil. The magnetic intensity at any point of this field is the resultant of the forces exerted by the poles of the magnet and those of the earth upon a free north pole at that point. By Coulomb's law these forces are inversely proportional to the square of the distance from the point to each pole. Since the compass needle indicates the direction of the resultant force and is, at every point of the paths described, tangent to the path, these paths represent lines of force.

Directions.—Suspend each of the bar magnets with its axis in a horizontal plane by means of the stirrup and fine wire, (See Fig. 39), and allow it to vibrate until it indicates clearly which is the north seeking pole. The north seeking pole should be marked with a file or pencil mark.

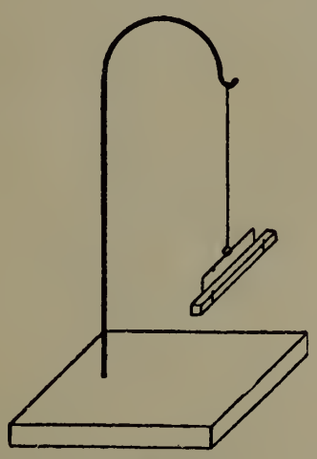


Fig. 39

Action Between Poles.—Suspend one of the magnets and bring it to rest. Then approach the north pole of the other to that of the suspended magnet and record the resulting action between them. Bring up an unlike pole and record the resulting action.

Effect of Strengthening or Weakening the Earth's Field.—Turn the magnet from its position of rest through a small

angle (about 30°) and allow it to vibrate *about a vertical axis through its center of suspension* in a horizontal plane. Note the time for five complete vibrations and calculate the mean period. Do the same for the second magnet. Next place the first magnet on the table near the second (suspended) magnet, like poles being adjacent. (See Fig. 40). Let the suspended magnet vibrate through a small arc, note the time of five complete vibrations and calculate the new period. (A second trial with the distance between magnets altered may be necessary to show the change in the period. Do not have the magnets so close together that

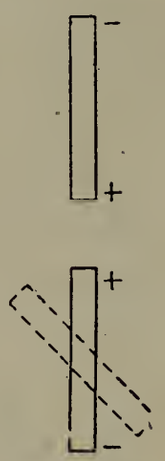


Fig. 40

the suspended magnet reverses its direction.) Repeat with unlike poles adjacent.

Astatic System of Magnets.—Suspend both magnets together, having like poles contiguous and find the mean period. Reverse one of the magnets and repeat. (If the reversed magnet has the stronger pole the system as a whole will reverse itself.)

Distortion of Earth's Field.—Place a sheet of paper with one edge parallel to that of the table and lay a bar magnet at the middle of the west edge, with its north pole pointing

north. Draw an outline of the magnet and label its ends properly + and −. Place a small compass at the middle of the + pole and move it about a quarter of an inch in the direction in which the needle points when it has come to rest, marking the direction of the needle by a short line, 1/8 inch long, behind the compass. Continue this operation until the edge of the paper or the other pole has been reached. Starting from six equidistant points about one side of each pole, proceed as before. (A semi-circle may be drawn about each pole with the pole as center, and six equidistant points laid off on each.)

From some point *c* on one of the lines of force which extends from one pole to the other *and near both poles*, draw lines in the direction of the force exerted by each of the two poles on a free north pole at that point, and on these lines lay off lengths which are inversely proportional to the squares

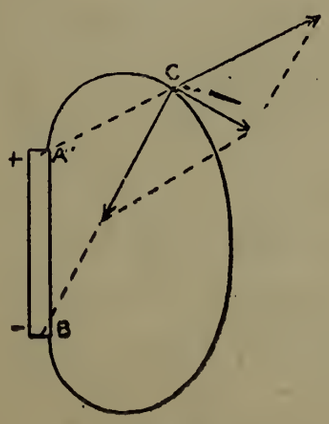


Fig. 41

of the respective distances from the point to the pole exerting the force. With these lines as component forces draw the resultant force and note its position with respect to the line of force. (See Fig. 41.)

On the reverse side of the paper proceed to trace the lines of force as before but with the magnet in the center of the paper, its + end west and its − end east .

Make a reduced copy of these fields of force on the data sheet.

QUESTIONS.—1. In the last part of the experiment why would it not be satisfactory to find the resultant force at a point distant from the magnet by this method?

2.—Two bar magnets when suspended as an astatic system have a period of a (=5.) seconds, but when the weaker one is reversed in the system the period is b (=3.) seconds. If both have the same dimensions find the ratio of their magnetic moments.

3.—A certain magnetic needle has a natural period of a (=4.) seconds. When suspended above and parallel to a bar magnet lying in the magnetic meridian, its period is b (=20.) seconds. How many vibrations per second will it make if the bar magnet is reversed?

4.—At a place where the earth's magnetic field has a horizontal intensity a (=0.2.) units, two parallel bar magnets each b (=12.) cm. long, are placed with their axes horizontal and in the magnetic meridian, with their unlike poles opposite each other and c (=16.)cm. apart. The poles pointing toward the south are d (= + 50.) and e (= − 25) units in strength respectively. Find the total force acting on a pole of strength f (= + 5) units at a point midway between the four poles of the magnets. State the direction of the force as well as its magnitude.

33. DETERMINATION OF H AND M

To determine the horizontal intensity of the earth's field and the magnetic moment of a magnet. (D. 375-384; G. 723, 724, 736; K. 491, 492, 501; W. 423-428.)

APPARATUS.—Large compass box with attached meter

rod, small compass, magnetometer box with suspension, magnet and holder.

Theory and Method.—If a magnetic needle is free to rotate about a point in the earth's field it will place itself in the magnetic meridian, making an angle with the horizontal, known as the angle of inclination or dip. If the needle is free to rotate in the horizontal plane only, it will be acted upon by the horizontal component H, of the earth's field and will set itself in the magnetic meridian. *By the horizontal intensity of the earth's field at any point we mean the number of dynes of force acting in the horizontal plane upon a unit north magnetic pole at that point.*

The period of vibration of a horizontally suspended magnet about its center is

$$T = 2\pi\sqrt{I/MH}$$

(See theory of experiment 32). From this equation we get

$$MH = \frac{4\ \pi^2\ I}{T^2}$$

We have also a method of determining M/H and are therefore able to calculate both M and H. Let a magnet whose period has been determined be placed with its axis perpendicular to the magnetic meridian and in a line with the center of a suspended magnetic needle, the distance between their centers being "a" (See Fig. 42). The earth's field, H, and that of the magnet, H' will be approximately uniform and perpendicular to each other at the magnetic needle, and will cause it to be deflected through an angle ϕ, its axis lying in the direction of their resultant, so that

$$tan\phi = H'/H$$

Let m be the pole strength and l the length of the magnet.

By Coulomb's law, the force between two poles of strength m and m' at a distance r apart, in air, is

$$F = \frac{mm'}{r^2}$$

Then, by definition, the intensity of a field due to a pole of strength m, at a distance r from the pole is m/r^2. If the suspended magnet is quite small compared with l and a, the

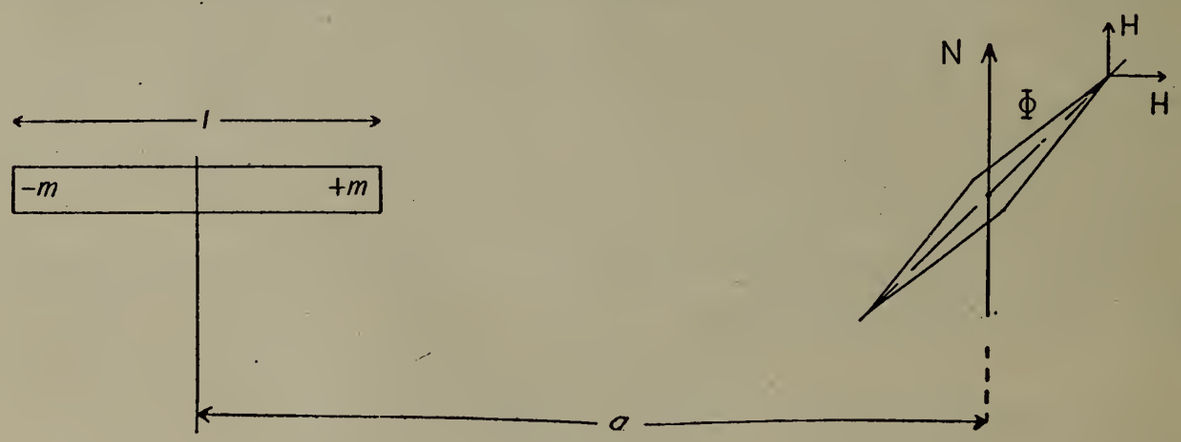


Fig. 42

approximate distance of the poles $+m$ and $-m$ from the poles of the suspended magnet are $(a - l/2)$ and $(a + l/2)$ respectively, and as H' is the field due to both $+m$ and $-m$,

$$H' = \frac{m}{(a - l/2)^2} - \frac{m}{(a + l/2)^2}$$

Combining these terms we have

$$H' = \frac{2mla}{(a^2 - l^2/4)^2} = \frac{2Ma}{(a^2 - l^2/4)^2}$$

where M is the magnetic moment of the magnet. As l is usually small compared with a, the terms $2a^2l^2/4$ and $l^4/16$ in the expansion of the denominator may be neglected and we have

$$H' = \frac{2Ma}{a^4} = \frac{2M}{a^3}$$

Substituting this value in the expression for $tan\phi$

$$tan\phi = (2M)/(a^3H) \quad \text{or}$$

$$\frac{M}{H} = \frac{1}{2} a^3 tan\,\phi$$

By combining this ratio with the expression for the product of H and M so that first one, then the other of these terms is eliminated, we get expressions for both H and M in terms of measurable quantities.

Directions.—*To determine MH.*

Place the magnetometer box with the glass sides facing north and south respectively. Suspend the magnet in the stirrup so that it lies in the horizontal plane when it comes to rest in the magnetic meridian. Then start it to oscillating through an angle of about 10°, preventing any motion of the center of the magnet by means of the wire stop at the side of the box. Determine the period of the motion from the time required for ten complete oscillations. Repeat this determination as a check. The magnets are marked with Roman numerals. Note from the bulletin board the mass, length and radius of the magnet you used, and calculate its moment of inertia and the value of MH. (See Appendix for formulae of moments of inertia).

To determine M/H.

The large compass box has a small magnetic needle with a long aluminum pointer attached perpendicularly to it, moving over a circular scale on a glass mirror. A meter stick is fastened symmetrically under the center of the box. The compass is turned until the needle hangs at rest, with the pointer in line with the meter stick, and at the same time the leveling screws are adjusted so that the pointer may swing freely about the center of the scale. Be sure the mag-

net is not close enough to affect the needle. Note on the scale the position of the ends of the pointer. To avoid errors due to parallax in taking this reading, place the eye vertically above each end, so that the pointer is seen covering its image in the mirror. Place the magnet symmetrically in the holder and put them on the meter stick with centers 25 cm. east of the needle. Note the direction of both ends of the pointer. Then reverse the magnet and when the needle has come to rest note the deflection of both ends from their original position. Repeat for 25 cm. west of needle. Do the same for 26 and 27 cm. Calculate the mean deflection in each case and the values of the tangent of this angle times the distance cubed. From the average of these products calculate the value of M/H, and combining with the value found for MH, determine both M and H.

QUESTIONS.—1. Explain how the following arrangements of the compass box would introduce errors in the measurements and how the errors were eliminated in each case:

a) The center of the scale did not coincide with the center of suspension?

b) The center of the meter rod was not under the center of suspension?

c) The poles of the bar magnet were not equidistant from the center of the magnet?

2. Find the least distance between magnet and needle in the above experiment for which the approximate formula $H'=2M/a^3$ can be used with a magnet $a(=10)$cm long without making an error greater than $b(=1.)\%$ in the value of H'.

3. A rectangular bar magnet having a length $a(=8.)$cm., width $b(=2.)$ cm., thickness $c(=1.)$ cm, and mass $d(=120)$.

grams makes $e(=10)$ vibrations per minute in a magnetic field whose strength is $f(=0.2)$ units. What is its pole strength? (See Appendix IV for moments of inertia.)

~ 4. A bar magnet with length $a(=10)$ cm. and pole strength $b(=20)$ units lies in the A position of Gauss with respect to a small compass needle, its center being $c(=100)$ cm. east of the needle, its *N.* pole toward the east. A similar magnet with length $d(=20)$ cm. and pole strength $f(=5.)$ units lies in the B position of Gauss, north of the needle, its axis east and west. If the needle is not deflected from the magnetic meridian, how far is it from the center of the second magnet?

34. GALVANOMETERS

To compare different types of galvanometers and to study the law governing the deflections of a tangent galvanometer. (D. 430-439, 442, 443, 449; G. 835-845, 850, 856; K. 600-603; W. 476-480.)

Apparatus.—Tangent galvanometer, astatic galvanometer, D'Arsonval galvanometer, resistance box, gravity cell, reversing switch, five connecting wires.

Theory and Description.—A galvanometer is an instrument for measuring the strength of an electric current.

By strength or intensity of current we mean the rate of flow of quantity of electricity. The quantity of electricity which passes through any cross section of a conductor in a given time is therefore equal to the current strength multiplied by the time.

The absolute unit of current strength in the C.G. S. system is defined as that current which, when flowing through a con-

ductor one centimeter long and bent in a circular arc of one centimeter radius, acts with a force of one dyne on a unit magnetic pole at the center of the arc. The practical unit of current strength is called *the ampere*, and *is one tenth as great as the absolute unit of current strength.*

The absolute unit of quantity of electricity is defined as the quantity of electricity which passes through any cross section of a conductor when one absolute unit of current strength flows for one second. The practical unit of quantity is called *the conlomb* and *is one tenth as great as the absolute unit of quantity.* The conlomb may also be defined as *the quantity of electricity which flows through any cross section of a conductor when a current strength of one ampere flows for one second.*

The essential parts of an ordinary galvanometer are a coil of wire through which the current flows, and a permanent magnet, either one of these being fixed, the other being suspended so that it is free to rotate in the magnetic field of force of the first.

The tangent galvanometer has a flat circular coil of wire, C, fixed in position, with its plane in the magnetic meridian so that when a current flows through it the magnetic field of force is at right angles to that of the earth. A small magnetic needle, N, is suspended by means of a silk fibre at the center of the coil. (See Fig. 43). When a current of electricity passes through the coil, the needle will be deflected from its north and south direction until its axis lies in the direction of the resultant of the two fields of force. If one absolute unit of current flows through our galvanometer coil having n turns of radius r, since the force varies as the length of wire and inversely as the square of the distance r, it will act on a unit magnetic pole at the center with a force $(2\pi rn)/(r^2) = (2\pi n)/r$ dynes. This force

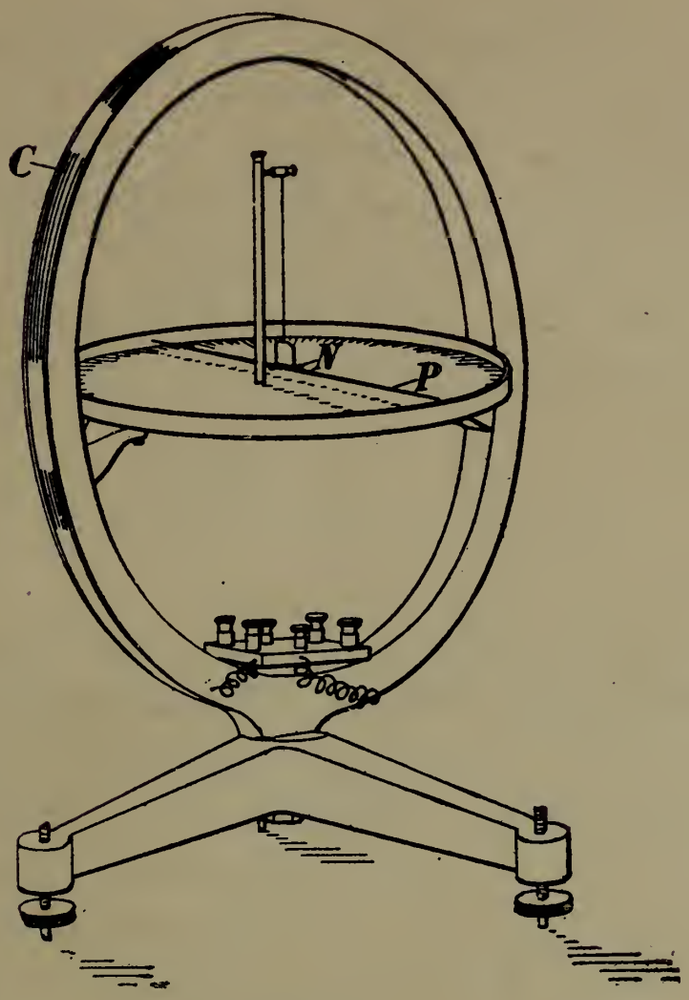


Fig. 43

$(2\pi n)/r$ dynes is called the galvanometer constant, G. The strength of field, F, at the center due to a current of I absolute units in the coil will be IG dynes, and will be practically uniform throughout the space in which the small needle moves. If the needle has a magnetic moment M and is deflected through an angle ϕ the moment of the deflecting couple will be $IGM\cos\phi$ and the moment of the balancing couple due to the earth's field of strength H will be $HM\sin\phi$. These moments must be equal when the needle is at rest or,

$$IGM\cos.\phi = HM\sin.\phi$$

and

$$I = \frac{H}{G}\tan.\phi = \frac{Hr}{2\pi n}\tan.\phi$$

H and G are constants for a given galvanometer in a given position and their ratio K is called the reduction factor of the galvanometer, the current strength in absolute units being K times the tangent of the angle of deflection. Since the practical unit of current strength is one tenth as great as the absolute unit, the *number* of practical units C will be ten times as great, or the current strength in amperes is

$$C = \frac{10H}{G} tan\ \phi = \frac{10Hr}{2\pi n} tan\ \phi$$

If the plane of the coil C is not in the magnetic meridian but makes an angle θ with it, the field of the coil, F, and the earth's field, H, will form an angle of $90° \pm \theta$ instead of being at right angles. (See Fig. 44.) F will have a component $F\ sin\ \theta$ in the same direction as H or opposite to it, and a component $F\ cos\ \theta$ at right angles to H. The rectangular components of the resultant force, R, which produces the deflection of the suspended magnetic needle, are then $H \pm F\ sin\ \theta$ and $F\ cos\ \theta$, the $+$ or $-$ sign depending upon the direction of F relative to that of H, that is upon the direction of the current. In other words, if the current flows in one direction the field R_1 will be the resultant of $F\ cos\ \theta$ and $H + F\ sin\ \theta$, where as if the current is reversed, the field R will be the resultant of $F\ cos\ \theta$ and $H - F\ sin\ \theta$. The resulting deflections will be unequal. (Both cases are shown in figure 44, the respective deflections being marked ϕ' and ϕ.) When the needle is deflected through an angle ϕ and comes to rest, the moments of the deflecting and the restoring couple will be equal. Hence

$$(H \pm F sin\theta)\ M sin\phi = F\ cos\theta\ M cos\phi,$$

and

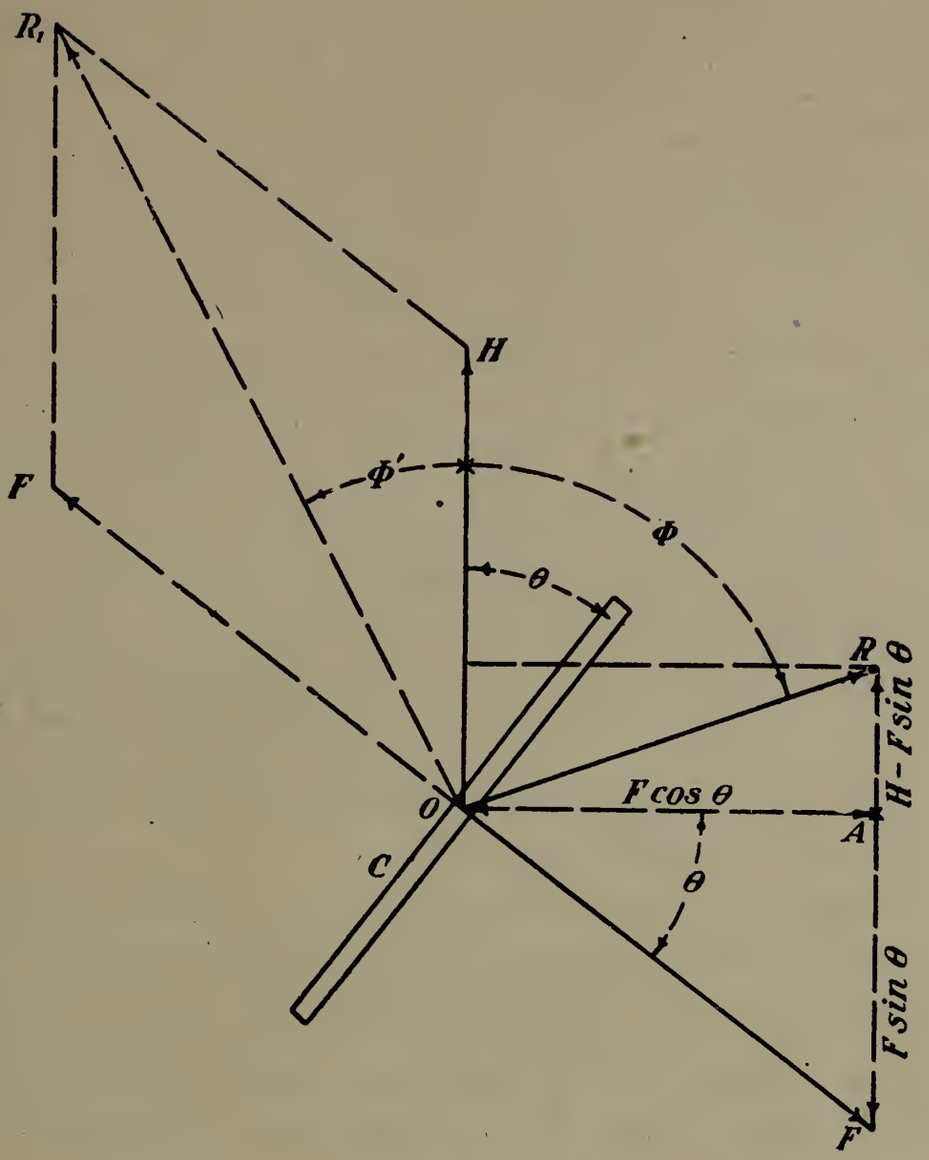


Fig. 44

$$\tan \phi = \frac{F \cos \theta}{H \pm F \sin \theta}$$

In order that the deflections may be equal for direct and reversed current, however, the plane of the coil must be in the magnetic meridian, or $\theta = o$. Under this condition $\sin \theta = o$ and $\cos \theta = 1$, hence the previous equation becomes

$$\tan \phi = \frac{F}{H} = \frac{IG}{H} = \frac{2\pi n I}{H r}$$

and

$$I = \frac{H}{G} \tan \phi = \frac{H\, r}{2\pi n} \tan \phi$$

as before.

In order to read the deflections of the needle (of the tangent galvanometer to be used) a long aluminum pointer, P, fastened at right angles to it, moves over a circular scale etched on glass. The lower side of the glass is silvered, forming a mirror so that errors due to parallax may be avoided by reading the position of the pointer when it and its image are in the same line of vision. It can be shown that the results are least affected by errors in these readings when the angle of deflection is about 45°. The galvanometers to be used have three coils the number of turns in each being different. That particular coil should be used in general, which with the available current strength gives a deflection of from 30° to 60°.

From the above equation for the current strength it is seen that the sensitiveness of such a galvanometer to weak currents is increased by increasing the number of turns of wire in the coil or by decreasing the radius of the coil or the directive force of the earth upon the suspended magnet. In the *astatic* galvanometer all three of these methods for increasing the sensibility are used. The needle system is astatic (see exp. 32), the number of turns of wire is large, and the coil is much smaller and has the shape of a flat oval, the conductors lying closer to the needles, so that they are in a stronger deflecting field. In some galvanometers each of the two needles is at the center of a coil, the two coils being wound in opposite directions, so that the deflecting force acting on the needles is in the same direction for both. In others the directive force of the earth is weakened by adjusting a permanent magnet above the galvanometer in such a position that it opposes the earth's field.

Both the tangent and the astatic galvanometers are protected from air currents by glass covers, but both are subject to the disturbing influences of surrounding magnetic fields.

In the D'Arsonval galvanometer, the small rectangular coil which carries the current is suspended by means of a phosphor-bronze or steel strip between the poles of a fixed permanent U shaped magnet. A fixed cylinder of soft iron projecting within the coil, serves to make the magnetic field in which the coil moves, stronger and more uniform so that within certain limits the deflections are directly proportional to the current flowing in the coil. The magnet is sufficiently strong to make the influence of the earth and of neighboring magnetic fields negligible, and the galvanometer does not have to be fixed in any definite position. In order to observe the small deflections of the coil accurately, a small plane mirror is fastened to it. A millimeter scale is fixed at about half a meter in front of the mirror and its image in the mirror is observed by means of a telescope fastened to the scale. The apparent deflection of the image of the scale as observed through the telescope is twice as great as the actual deflection of the coil and mirror. (See Fig. 22). The telescope may be dispensed with, if the plane mirror is replaced by a concave one. In this case the scale is fastened to a plate of ground glass or other semi-transparent substance, and an illuminated slit is placed at its center. The distance between scale and mirror is adjusted until the image of the slit or of the lamp filament is sharply focussed on the ground glass, so that the deflections of the image may be observed.

Either of the above two types of galvanometers may be designed for use in three different kinds of measurements.

a) We may wish to measure a current of definite strength which produces a steady deflection of the suspended

coil or needle. The tangent and the D'Arsonval galvanometer are frequently used for this purpose. The galvanometer should be aperiodic or dead-beat; that is, its suspended needle or magnet should come to rest quickly, without vibrating back and forth. In the first type of galvanometer described, this damping is produced by surrounding the needle (usually of an inverted U shape) with a mass of copper. In the D'Arsonval type the damping is produced by winding the coil on a metal frame or placing it in a metal tube. In either type eddy currents are induced in the solid metal and prevent vibrations. (See Exp. 43).

b) We may wish to detect the presence of very small currents, in which case a very sensitive galvanometer such as an astatic or a D'Arsonval is needed. This is the case in all measurements where a null method is used; that is, where the apparatus is to be adjusted until no current flows. (See Exp. 36, 37, 38, etc.)

c) We may wish to measure, not the strength of a current, but the total quantity of electricity flowing through the galvanometer during a short interval of time, for instance, the discharge of a condenser, or an induced current. (See Exp. 43.) A galvanometer adapted to such measurements is called a ballistic galvanometer. It should have as little damping as possible, and a long period of vibration so that the needle or coil does not move appreciably from its position of rest, before all of the electricity has passed through the coil. Under these conditions the quantity of electricity is proportional to the sine of half the angle of the first swing (or throw.)

The resistance box contains several coils of wire of different lengths and cross sections, each coil having a definite electrical resistance of a whole or fractional number of ohms. (See Fig. 45.) The coils are wound non-inductively by re-

turning the wire upon its own path, and each end of a coil is connected to a separate brass block on the upper surface of the box. Two coils are connected to the same block in

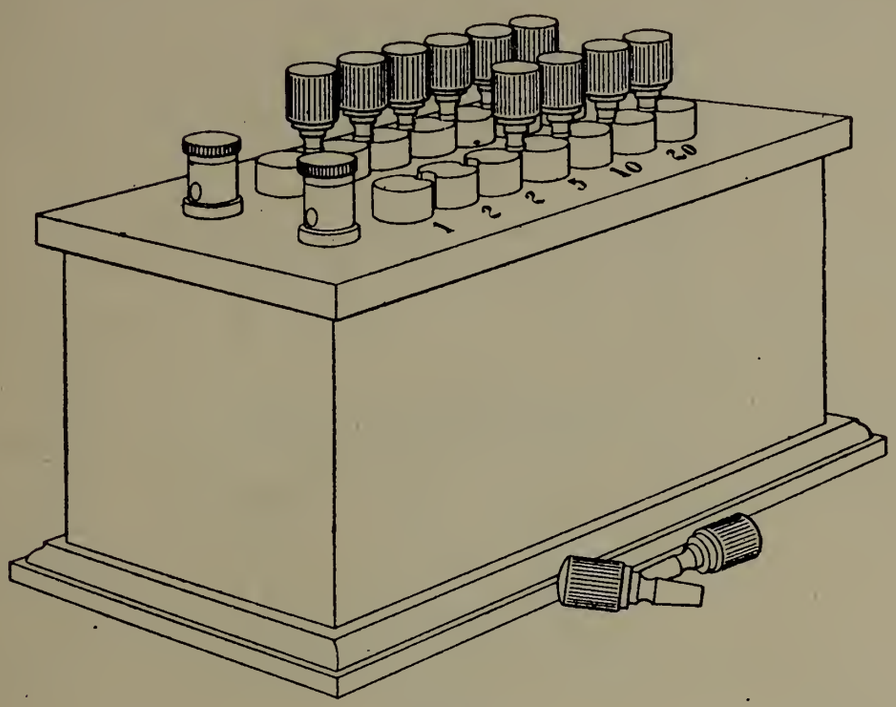
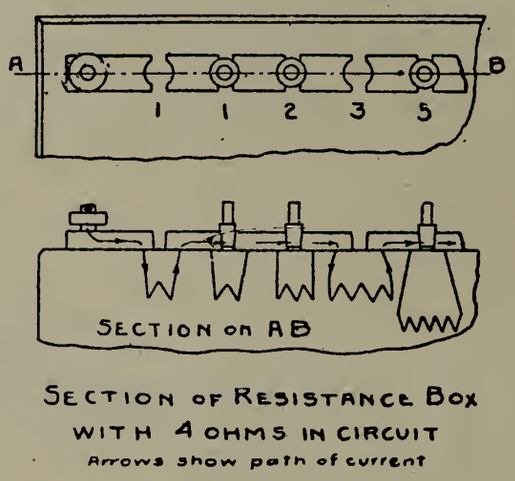


Fig. 45

such a way that a current of electricity may be passed successively from one coil into the other, and the resistance of the circuit may be varied by including more or less of these coils. But when a brass plug is inserted between two blocks, the current will pass partly through the coil, and partly through the plug. (See Fig. 46.) The cross section of plugs and blocks is made so large that their combined resistance in parallel with the coil is negligible (See Exp. 36) and for all practical purposes that coil is cut out of the circuit. The resistance of each coil, in ohms, is marked beside the gap between the blocks to which the ends of the coil are attached. Unless there is very good contact between the plugs and blocks, the resistance to the flow of current through these surfaces of contact will be appreciable and may cause considerable error in the total resistance.

It is advisable to use a few large resistance coils rather than a large number of small ones.

DIRECTIONS.—Adjust the level of the tangent galvanometer so that the pointer swings freely over the whole scale and turn the galvanometer until the plane of the coil is in line with the magnetic *needle* when at rest. Remove the



SECTION OF RESISTANCE BOX
WITH 4 OHMS IN CIRCUIT
Arrows show path of current

Fig. 46

short circuiting coil from the gravity cell and connect the apparatus as shown in the diagram, Fig. 47, G being the galvanometer, B the galvanic cell, R the resistance box, and S the switch.

Note the position of both ends of the pointer when the switch is open in which case the resistance of the circuit may be considered infinitely great for all practical purposes. Close the switch and when the needle comes to rest again note the position of the pointer. Then reverse the current and note the position of the pointer. If the mean deflection is not the same in both cases it means that the plane of the coil is not in the magnetic meridian and it must be turned toward the magnet (not the pointer) in the position of smaller deflections, until the deflections become equal for reversed currents. The box resistance during this trial has been zero, and when the *deflections* are equal they should

be recorded for both ends of the pointer and for both direct and reversed currents. The corresponding deflections for 1, 2, 5, and 10 ohms of box resistance in the circuit should be observed and the mean deflection calculated. This box resistance is only a part of the total resistance of the circuit, the latter including the resistance of cell, galvanometer, and connecting wires. To determine this resistance exclusive of the box resistance, plot a curve having for ordinates the *cotangents* of the angles of deflection and for abscissae, the corresponding box resistances. The intercept of this curve on the axis of abscissae between the origin and curve gives the required resistance to the same scale as the plotted box resistance. Calculate the total resistance in each case.

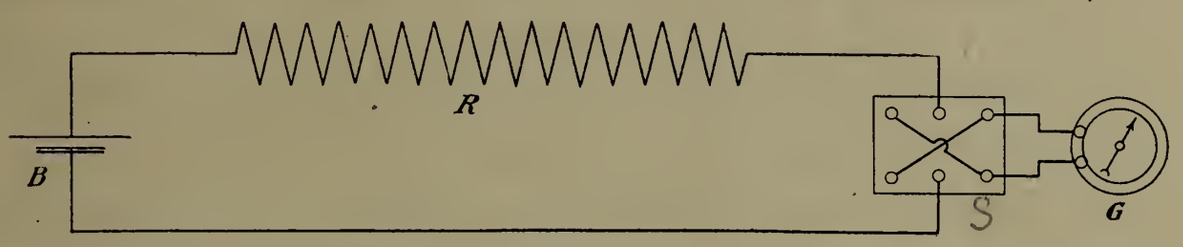


Fig. 47

According to Ohm's law, the current strength in the circuit equals the electromotive force (*e. m. f.*) in the circuit divided by its *total* resistance. (See Exp. 36.) Note the *e. m. f.* of the cell, and calculate the current strength and the reduction factor K of the galvanometer in each case.

Examine one of the D'Arsonval galvanometers on the wall and include a sketch of the essential parts in your written report. Do the same for the astatic galvanometer.

Questions—1.—*a*) With the data obtained in the above experiment plot a curve between corresponding values of current strength (as abscissae) and galvanometer deflections.

b) On the same sheet of coordinate paper plot a second curve between corresponding values of current strength and *tangents* of the angles of deflection.

2.—Explain why the intercept of the above curve gives the resistance of the circuit exclusive of box resistance.

3.—*a*) What errors due to the lack of symmetry in the apparatus, are eliminated by the method of taking deflections described above.

b) Would the readings be affected if the pointer were bent, or if it were not fixed at right angles to the magnetic needle? Give reasons for your answers.

4.—The coil of a tangent galvanometer with a steady current flowing through it is placed at right angles to the magnetic meridian. The needle when set in vibration makes $a(=5)$ oscillations per second, but when the current is reversed it makes $b(=3)$ oscillations per second.

a) How does the field of the coil at its center compare with that of the earth?

b) What will be the deflection of the needle if the plane of the coil is placed in the magnetic meridian?

35. OHM'S LAW AND POTENTIAL DROP.

To test Ohm's law in a simple circuit, and to show that the potential drop in a conductor is proportional to the resistance of the conductor. (D. pp. 395, 403, 411; G. pp. 866-870, 953; K. pp. 415, 433, 434.)

APPARATUS.—D'Arsonval galvanometer with shunt box, resistance box (10,000 ohms), high resistance wire with sliding contact, gravity cell, reversing switch, and seven connecting wires.

THEORY AND DESCRIPTION.—*We may define the electrical potential at a point as the work done in bringing unit*

positive quantity of electricity from an infinite distance up to the point in question. It is shown in books on advanced physics that the work required to bring unit quantity of electricity from an infinite distance to any point is the same whatever path may be followed. In bringing this quantity up to a second point we may therefore follow a path which passes through the first point. It follows that *the difference of potential between two points in an electric circuit is the work done upon unit quantity of positive electricity in bringing it from the point of lower potential to the point of higher potential; or it is the work done by unit quantity of positive electricity as it flows from the point of higher potential to the point of lower potential.* A "current" shall be understood to mean a flow of positive electricity. (Unit quantity of electricity and unit current strength were defined in experiment 34.)

Unless there is a difference of potential between two points of a circuit no current can flow between them, and positive electricity always flows from a point of higher to a point of lower potential. The difference of potential between two points is therefore often spoken of as *the potential drop* between them.

When a current of electricity flows in a simple closed circuit consisting of a source of electricity (such as a galvanic cell or dynamo) and a conductor which connects the terminals of the source, the current after flowing from one terminal through the external conductor must flow back to the first terminal through the conductors within the source, (the plates and liquid of the cell, or the wire coils of the dynamo armature), and there must be a potential drop within the source as well as through the external circuit. The electromotive force of the source may be defined as the sum of the external and internal drops of

potential in the circuit; or in general *the electromotive force in a circuit is the total drop of potential (or the sum of all the potential differences) met with in passing from one terminal of the source of electricity through the complete circuit and back to the same terminal.*

The absolute unit of potential difference or electromotive force, in the C. G. S. system, exists between two points in a circuit when one erg of work must be done in order to move unit quantity of positive electricity from the point of lower to the point of higher potential. The practical unit of potential difference or electromotive force is called *the volt* and *is* 10^8 *times as large as the absolute unit.* The volt may also be defined as *the potential difference existing between two points in a circuit when one joule* (10^7 *ergs*) *of work is done by one conlomb of electricity (one ampere flowing for one second) as it flows from the point of higher potential to the point of lower potential.*

Ohm's law states that when a constant electromotive force is impressed upon a closed circuit, there is a constant ratio between the electromotive force and the resulting current strength. This constant ratio is called *the resistance* of the circuit. Ohm's law is more commonly stated in this form: The strength (I) of the electric current flowing in a closed circuit equals the total electromotive force (E) in the circuit divided by its total resistance (R), or in symbols

$$I = \frac{E}{R}$$

The total electromotive force is the summation of the *e. m. f.'s* of all sources of electricity in the circuit, each with its proper sign. (See Exp. 38.) The total resistance of the circuit is the summation of the resistances of all the conductors in the circuit, the method of summation depend-

ing upon the manner in which the conductors are connected. (See Exps. 36 and 37.)

The absolute unit of resistance in the C. G. S. system is the resistance of a conductor in which a current strength of one absolute unit will flow when the absolute unit of potential difference exists between its ends. The practical unit of resistance is called *the ohm* and *is* 10^9 *times as large as the absolute unit.* The ohm may also be defined as *the resistance of a conductor in which a current strength of one ampere flows when one volt of potential difference exists between its ends.*

Ohm's law is true not only for any closed circuit as a whole, upon which a constant *e. m. f.* is impressed, but it holds for any part of the circuit as well. The potential drop between any two points of the circuit must equal the resistance between these two points multiplied by the strength of current flowing between them, and the sum of all the potential drops met with in passing completely around the circuit, must equal the total electromotive force in the circuit.

In order to test Ohm's law we may connect a source of constant *e. m. f.* such as a gravity or storage cell in a circuit containing a box with variable resistance, and a galvanometer arranged to measure the current strength in the circuit. (See Exp. 39.) If the box resistance is made very large in comparison with that of the cell and galvanometer circuit, the latter may be neglected. On changing the box resistance the current will change in such a way that the product of current strength and box resistance will remain equal to the electromotive force. When the galvanometer is arranged so that its deflections are directly proportional to the current strength producing

them, the product of box resistance and corresponding deflections will be constant also.

To show that the potential drop through different parts of a circuit is directly proportional to the resistance of these parts we may connect a long uniform high resistance wire in circuit with a source of constant current (gravity or storage cell), and arrange a galvanometer to show the potential drops through different lengths of the wire. (See Exp. 39.) The ratio of the potential drop through any section of the wire and the resistance of this section will be constant. Since the deflections of the galvanometer are directly proportional to the potential difference measured and since the resistance of any section of a wire of homogeneous material and uniform cross-section is directly proportional to the length of this section, the ratio of deflections and corresponding lengths will be constant also.

DIRECTIONS.—*a) To show that for a constant e. m. f, in circuit the current is inversely proportional to the resistance.* Connect the apparatus as shown in figure 47, where *B* is a gravity (or storage) cell, *R* a resistance box with at least 10,000 ohms, and *G* a D'Arsonval galvanometer with a 1 ohm shunt (See Exp. 36.), connected in the circuit by means of a reversing switch. The deflections of the galvanometer, if they are not too large, will be proportional to the current strength in the circuit. Remove the necessary plugs from the resistance box to insert 10,000 ohms in the circuit. Close the switch and note the deflection of the galvanometer. Reverse the current by means of the switch and again note the deflection. Calculate the mean of the two deflections and the product of this mean and the box resistance. Change the resistance from 10,000 to 3,000 ohms in steps of 1,000 ohms, and note the deflections for direct

and reversed current in each case. Calculate the products of box resistance and corresponding mean deflection for each case. Compare these products and see that they are practically constant.

b) To show that for a constant current flowing in a circuit the potential difference through any section of the circuit is directly proportional to the resistance of this section. Connect the apparatus as shown in figure 56, where B is a gravity (or storage) cell connected through a reversing switch to the ends of a high resistance wire R. This wire is of homogeneous material and uniform cross section, 10 meters long, and is stretched back and forth in one meter lengths on a board (See Fig. 53.) The D'Arsonval galvanometer G with a 10 ohm shunt S is connected through a 10,000 ohm resistance R_1 to the end D of the long wire R. The other terminal of the galvanometer is connected to a sliding contact which may be placed at any point on the long wire. The deflections of the galvanometer when so arranged, are proportional to the potential drop through the section of wire across which it is connected. Place the sliding contact at 50 cm from the end D of the wire and note the deflections of the galvanometer when the switch is closed, first with the current flowing in one direction, then with the current reversed. Calculate the mean deflection and the ratio of this mean to the length of wire (50 cm.) Do the same with the sliding contact at 250, 450, 650, 850 and 1000 cm from D respectively and calculate in each case the ratio of the mean deflection to the corresponding length of wire. Compare these ratios and see that they are practically constant.

Questions, etc.—1.—*a*) What kind of curve would you obtain if you plotted total resistance (in part *a*) of the

above experiment) as abscissae and corresponding deflections as ordinates.

b) Plot a curve having lengths of wire (in part *b* above) as abscissae and corresponding deflections as ordinates.

2.—A storage battery with an internal resistance $a(=0.5)$ ohms sends a current $b(=1.5)$ amperes thru a resistance coil of $c(=2.3)$ ohms in series with an incandescent lamp whose resistance is $d(=1.8)$ ohms. Find the potential drop in the lamp and the *e. m. f.* of the battery.

3.—If in part *a*) of the above experiment the products obtained were used in calculating the *e. m. f.* of the cell, the deflections being $a(=1.)$ mm. per $b(=0.001)$ ampere, the resistance of the cell $c(=10)$ ohms and the combined resistance of galvanometer and shunt $d(=0.98)$ ohm, find the % error in the calculated *e. m. f.* for a box resistance of $e(=3000)$ ohms.

4.—A battery having an *e. m. f.* $a(=5.6)$ volts and internal resistance $b(=2.)$ ohms sends current through a wire of length $c(=20)$ meters and resistance $d(=38)$ ohms. What is the potential drop through $e(=5)$ meters of the wire?

36. THE WHEATSTONE SLIDE WIRE BRIDGE.

To determine the resistance of several coils of wire, separately and in combination, by means of the Wheatstone bridge, and to study the laws for determining the total resistance of various combinations of separate resistances. (D. 442, 443, 449-452, 456; G. 853-856, 922, 923; K. 642-645, 651, 652; W. 480, 485-488.)

APPARATUS.—Wheatstone bridge, D'Arsonval galvanometer, galvanic cell, contact key, sliding contact, two resistance boxes with known and unknown resistance coils respectively, and nine connecting wires.

Description and Theory.—The slide wire bridge for comparing resistances consists of a wire w, usually one meter long, of uniform high resistance, stretched on a board and soldered at the ends (a and b)to two heavy copper angle bars. (See Fig. 48). A third straight bar is fastened to the board so that there is a gap between it and each angle bar. A resistance box R is connected to the bars at one of these gaps, the unknown resistance X which is to be determined, is connected at the other gap. At a point midway between the two a sensitive galvanometer G is joined to the straight

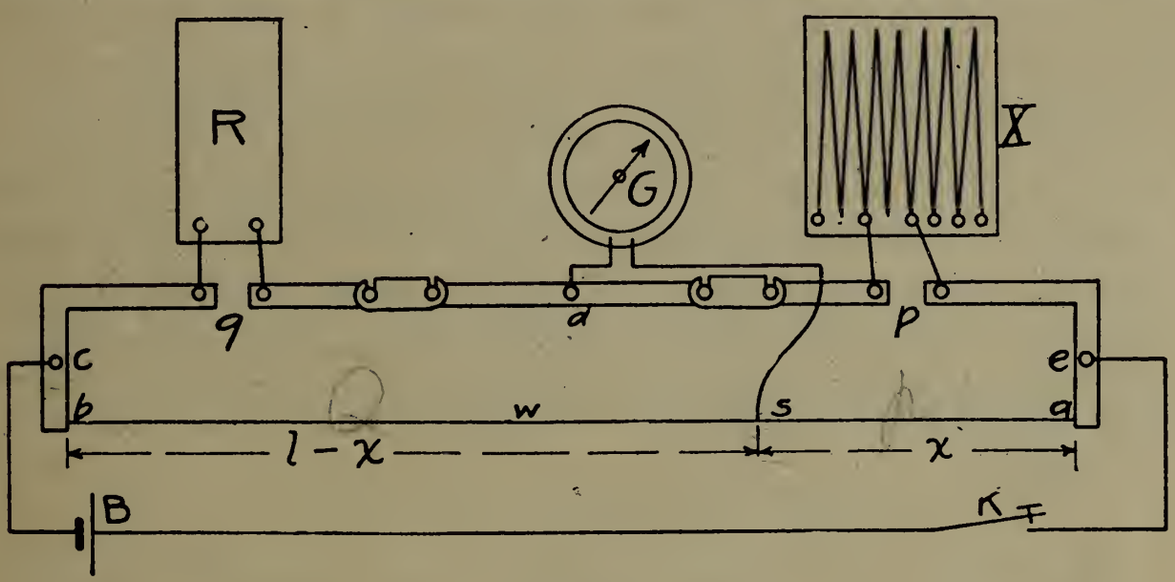


Fig. 48

bar and to a sliding contact key S which may be moved along the wire w. A galvanic cell B is connected to the two angle bars at c and e with a contact key K in the circuit. The resistance boxes and bars together with the wire w form a divided circuit so that when the key K is closed a current of electricity flowing from B to c would divide, part of it flowing through w and the remainder flowing through R and X and thence from e back to the cell. If the key S be closed there will in general be a deflection of the galvanometer, but on moving the sliding contact along the wire w a point is reached where, when K and S are closed, there is

no deflection of the galvanometer, which indicates that there is no current flowing through it.

Ohm's law holds for any part of a circuit as well as for the whole circuit (See Exp. 35) and according to this law the potential drop (instead of the electromotive force) through any part of the circuit equals the current strength times the resistance of that part of the circuit. If there is no current through the galvanometer circuit, there can be no potential drop through it, or in other words, the points d and S are at the same potential. Now let the total length of wire be l, let the length aS be x and its resistance be P, then the length of bS will be $l-x$ and its resistance say Q. Further let the current strength in R be I' and that in w be I''. Since no current flows off through the galvanometer circuit, the current in X will be the same as that in R. Then if d and S are at the same potential, the potential drop from c to d must equal that from c to S and the drop from d to e must equal that from S to e.

That is

$$RI'=QI'' \text{ and } XI'=PI''$$

Dividing the second equation by the first we have

$$\frac{XI'}{RI'}=\frac{PI''}{QI''} \text{ whence } X=\frac{P}{Q}R.$$

But the resistance of a conductor of uniform cross section and homogeneous material is directly proportional to its length (See Exp. 37), hence the ration P/Q equals the ratio $x/(l-x)$, so that

$$X=\frac{x}{l-x}R.$$

The unknown resistance may therefore be calculated from the measurable lengths of wire and the known resistance, neglecting that of the heavy bars.

It may be shown that in this form of the Wheatstone bridge the calculated result is least in error when $X=R$ and $P=Q$. It is therefore desirable to adjust R until the balance position of S (where there is no deflection of the galvanometer when both K and S are closed) is as near the middle of the bridge wire as is possible with the available known resistances.

When several resistances r_1, r_2, r_3, etc., are connected end to end in such a way that the current may pass successively from one into the other, they are said to be in *series*. Let the potential drop in r_1 be v_1, that in r_2 be v_2,

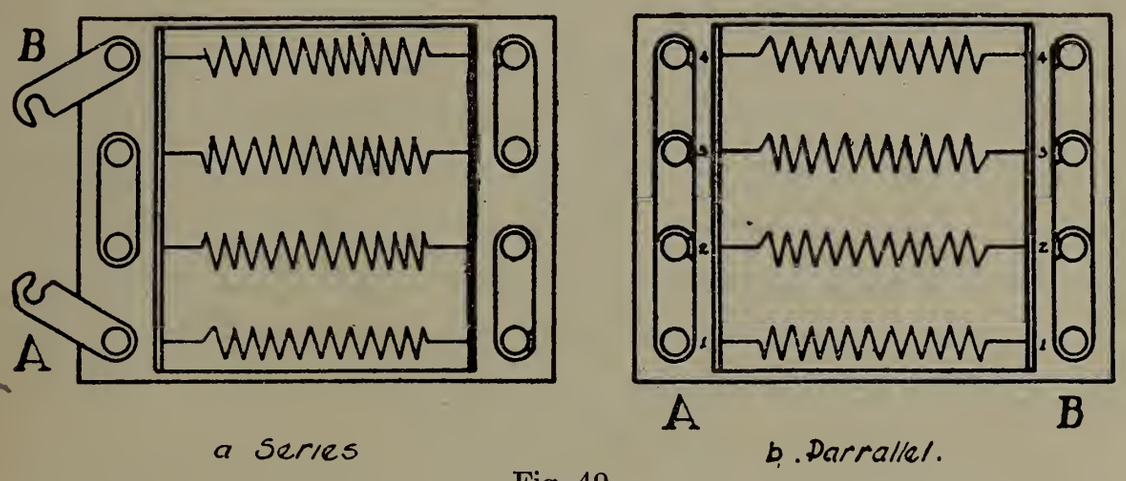


Fig. 49

etc., and let the total drop through all of them be V. Then $V=v_1+v_2+v_3+$ etc., and since the current strength I is the same throughout the circuit, we have on dividing through by I

$$\frac{V}{I}=\frac{v_1}{I}+\frac{v_2}{I}+\frac{v_3}{I}+\text{ etc.},$$

but by Ohm's law $V/I=R$, the total resistance, $v_1/I=r_1$, etc. Therefore this equation may be written

$$R=r_1+r_2+r_3+\text{etc.}$$

or in words, *the total resistance of several conductors joined in series is equal to the sum of their separate resistances.* This holds for any number of resistances in series. In figure

49a, if the current enters by the terminal A and leaves by the terminal B, it must pass through the four coils in series.

If several resistances are connected so that they form a divided circuit, by connecting one end of each together and to one terminal of the source of current, the other ends all being connected together and to the other terminal, the current will pass simultaneously through all of them, and they are said to be in *parallel*, (or in multiple arc.) In figure 49*b*, if the current enters by the terminal A and leaves by the terminal B, it will pass through the four coils in parallel; that is, it will divide and part of it will pass through each of the four branches. Those terminals of the resistances which are connected together must be at the same potential, or in other words, the drop of potential V in any one resistance must equal that in any other resistance and also that in their combined resistance R. Let the current strengths in resistances r_1, r_2, r_3, etc., be i_1, i_2, i_3, etc., respectively. The total current, I, will divide among the various branches so that

$$I = i_1 + i_2 + i_3 + \text{etc.}$$

or by Ohm's law, since

$$I = \frac{V}{R}., \; i_1 = \frac{V}{r_1}. \; i_2 = \frac{V}{r_2}. \text{ etc.}$$

$$\frac{V}{R} = \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3} + \text{ etc.}$$

Dividing through by V gives

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \text{ etc.}$$

or *the reciprocal of the total resistance equals the sum of the reciprocals of the separate resistances.* The reciprocal of the resistance of a conductor is called its conductance, and the law may be stated as follows:—*When conductors are joined in parallel their joint conductance is the sum of their separate conductances.* Conductance is expressed in terms of a unit called the *mho* (the reverse of ohm.) From the equation given it is seen that for two conductors in parallel, the last equation may be written in the form

$$R=\frac{r_1\,r_2}{r_1+r_2}, \text{ for three, } R=\frac{r_1\,r_2\,r_3}{r_1\,r_2+r_2\,r_3+r_3\,r_1} \text{ etc.}$$

It should be noted that the total resistance of a number of conductors in parallel is always less than the resistance of any one of them alone.

Shunts—Since for parallel circuits the drop of potential is the same through all of the branches, in the case of two branches

$$V=IR=i_1r_1=i_2r_2$$

Therefore

$$i_1 : i_2 = r_2 : r_1$$

or in words, the currents in branched circuits are inversely proportional to the resistance of the branches, so that the larger part of the total current flows through the smaller resistance and vice versa.

From the equations obtained for R and V we derive the following expressions for i_1 and i_2:

$$i_1=IR/r_1=Ir_1r_2/(r_1+r_2)r_1=Ir_2/(r_1+r_2)$$
$$i_2=IR/r_2=Ir_1r_2/(r_1+r_2)r_2=Ir_1/(r_1+r_2)$$

Therefore

$$i_1 : I=r_2 : (r_1+r_2)$$

and

$$i_2 : I = r_1 : (r_1 + r_2)$$

or in words, *the current in one branch of a parallel circuit is to the total current as the resistance of the other branch is to the sum of their resistances.*

The sudden rush of a strong current through a galvanometer causes a sudden large deflection of the movable system, which is likely to break the suspension. A large current may also overheat the coil and injure the insulation of the wire, etc. To avoid these effects a low resistance is joined in parallel with the galvanometer, so that the larger part of the current is diverted from it. Such a low resistance is called a shunt, and sensitive galvanometers are frequently supplied with three shunts of such resistance that only 0.1, 0.01, or 0.001 of the total current may pass through the galvanometer when they are joined in parallel with it. If S be the resistance of the shunt, G that of the galvanometer, I the total current and Ig that through the galvanometer, we see from the above theory of parallel circuits that

$$I_g : I = S : S + G$$

or

$$I_g = I \frac{S}{S + G}$$

so that for values of S that are small compared with G, the galvanometer current is a small fraction of the total current.

The shunt box shown in Figure 50, is connected to the galvanometer by means of the binding posts A and B. Any one of the three shunt coils may be placed in parallel with the galvanometer coil by putting the plug in the gap between that coil and the block of metal to which the binding post B is connected.

Directions.—Connect the apparatus as shown in the diagram (Fig. 48). The unknown resistance consists of four coils, each having its ends joined to binding posts at opposite sides of the box. (See Fig. 49). Connect the coil marked No. 1 with the bridge and, with the smallest value

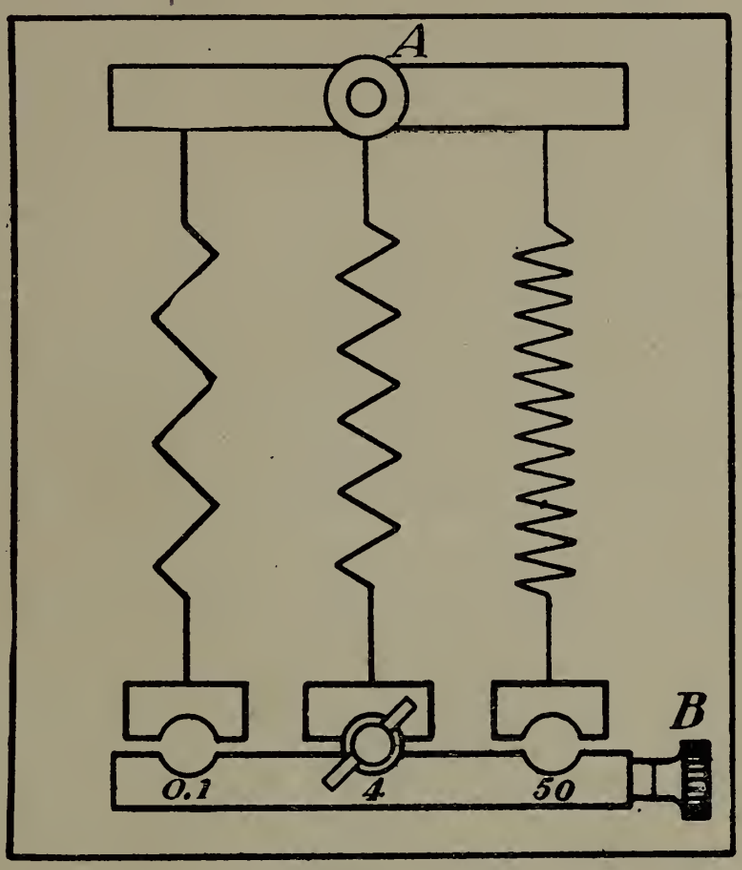


Fig. 50

of R in circuit and the key S at the middle of the bridge wire, note the deflection of the galvanometer when both K and S are closed. K should always be closed first, and since an open circuit cell is used it should not be kept closed longer than necessary. S should never be pressed down on the wire while it is being moved. The same pressure should be used on the keys at all times.

Increase R and note whether the direction of the deflection is opposite to what it was before. If not, try a still greater value of R. In this way find two values of R for which the deflections are in opposite directions. The value

of X must evidently lie somewhere between these two values of R, and when the deflection is in one direction R is too small, when in the opposite direction R is too large. With these relations fixed in mind, vary R by smaller and smaller steps until the minimum deflection is reached, then shift the contact key S to one side or the other in steps of a half millimeter or less, until there is no further decrease in the deflection of the galvanometer when both K and S are closed. S should be kept as near the middle of the bridge wire as possible. During all of the preliminary trials a low value shunt should be connected across the terminals of the galvanometer but when the balance position has been approximately determined the shunt may be removed in order to make the final adjustment. The balance position may also be determined by sliding S from one end of the bridge toward the center until equilibrium is obtained, then from the other end of the bridge toward the center until no deflection is obtained. If the two balance positions do not coincide, their mean should be taken as the balance position. As a check X and R may be interchanged and balance again obtained. Of course x and $l-x$ should interchange their values also. Note the value of R, x and l and repeat the performance with each of the other three coils. Then do the same with all the coils in series, with 1 and 2 in parallel, and finally with all of them in parallel. Calculate the resistance and conductance in each case, and show that the laws for parallel and series connection of resistances hold.

QUESTIONS.—1.—From the general expression for the total resistance of a number of resistances connected in parallel, show that the total resistance of n coils in parallel, each of resistance r will be r/n.

2.—What resistance would be required in series with

a galvanometer whose resistance is $a(=200)$ ohms, in a circuit with an electromotive force of $b(=1)$ volt, in order that the current through the galvanometer may be $c(=0.0001)$ ampere?

3.—If in place of the resistance calculated in question 2 we put $d(=160)$ ohms in series with the galvanometer, what resistance used as a shunt to the galvanometer, would give the same galvanometer current with the same electromotive force?

4.—*a)* Given three resistance coils A, B and C, with resistances $a(=1)$, $b(=2)$ and $c(=3)$ ohms respectively, describe (or sketch) the eight possible combinations of all three coils (arranged in series, or parallel, or combinations of the two)which will give a different total resistance.

b) Calculate the total resistance in each case.

37. THE POST OFFICE BOX BRIDGE

To determine the specific resistance of several wires and the variation of their resistance with length and cross section (D. 442-447, 456; G. 923, 844, 845, 850, 856; K. 641, 642, 646-649, 651; W. 480-484, 486, 487.)

Apparatus.—Post office box bridge, dry cell, D'Arsonval galvanometer, board with unknown resistances of manganin and German silver, six connecting wires.

Theory and Description.—*The resistance of a conductor may be defined as the ratio of the e. m. f. applied to its ends and the resulting current strength flowing through it.* It may be shown that the resistance of a conductor is directly proportional to its length l, and inversely proportional to its cross section A, and that it depends upon the material of the conductor, or in symbols

$$R = p\ (l/A)$$

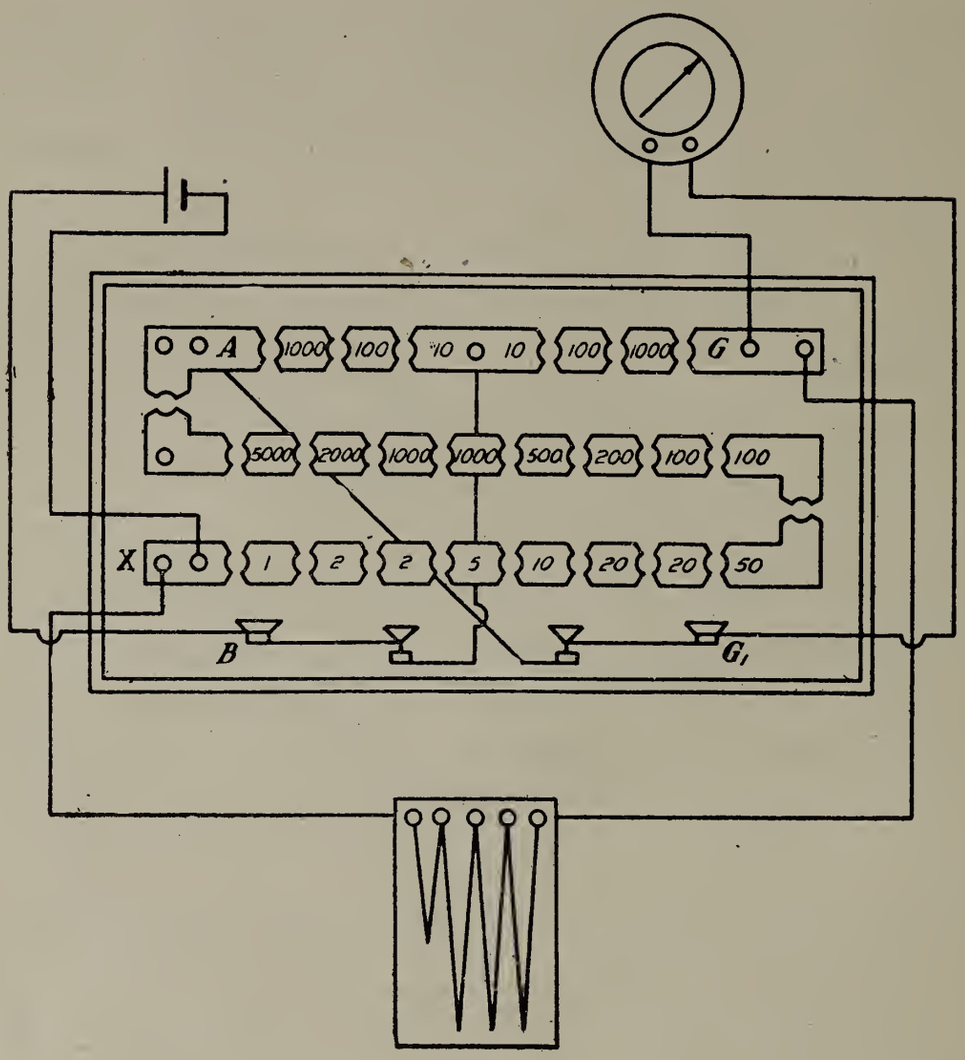


Fig. 51

The factor p is a constant for any one substance at a given temperature, and is called the specific resistance of the substance, or its resistivity. If both l and A are unity, R will equal p, hence *the specific resistance of a substance* may be defined as *the resistance of a conductor of that substance having a length of one centimeter and a cross section of one square centimeter.*

The Post Office box is a compact form of the Wheatstone bridge, in which there are two identical sets of resistance coils (A to G in Fig. 51) generally of 10, 100 and 1000 ohms, called the ratio arms of the bridge, and the usual set of known resistance coils R, (between A and X) with two contact keys, all contained in the same box. The con-

nections are somewhat different from those of the slide wire bridge. There is a fixed connection from the junction of the two ratio arms to the left hand key, and another from the block A to the right hand key, by means of wires within the box. The galvanic cell is to be connected to the binding posts B and X, the galvanometer to the binding posts G and G_1, and the unknown resistance X between the binding posts marked X and G.

If we let R_1 be the ratio arm adjoining G and R_2 the one adjoining A, and represent R_1, R_2, R and X (in the order given) as the sides of the diamond shaped figure which is the usual diagrammatic representation of the Wheatstone bridge, we find on tracing out the connections in Fig. 51, that the galvanic cell is connected between the junction of the two ratio arms and the junction of the known and unknown resistances; and that the galvanometer is connected across the two ratio arms. It is readily shown by applying the same train of reasoning to this diagram as was used in the case of the slide wire bridge, that for values of R, R_1, and R_2 which balance the bridge (give zero deflection of the galvanometer) the unknown resistance may be calculated from the proportion

$$X : R = R_1 : R_2$$

From this proportion it may be seen that if X be greater than R, then R_1 must be greater than R_2, and vice versa.

Since $X = R \times R_1/R_2$ the smallest unknown resistance which can be measured with this bridge is the smallest fraction R_1/R_2 (which can be formed with the fixed values of R_1 and R_2) of the smallest of the known resistances R. In other words if R_1 and R_2 are set at 10 and 1000 ohms respectively, and the smallest of the known resistance coil is 1 ohm, the unknown resistance can be determined to

1/100 of an ohm, whereas if R_1 and R_2 are set at 10 and 100 ohms respectively, the unknown resistance may be determined to only 1/10 ohm.

In any form of Wheatstone bridge there are six resistances to be considered, namely the known and unknown resistances, that of the galvanometer, that of the cell, and those of the ratio arms (or of the two sections of wire in the slide wire bridge.) In books on advanced Physics it is shown that the most sensitive arrangement of the bridge is to have the greater of the two resistances, that of the galvanometer and that of the galvanic cell, connected between the junction of the two larger remaining resistances and the junction of the two smaller ones. In general the galvanometer has a greater resistance than the cell, and since X and R_1 are either greater or less than R and R_2 respectively the arrangement of the bridge shown in fig. 51 is the most sensitive one.

DIRECTIONS.—Connect the apparatus as shown in the diagram. The unknown resistance consists of four wires. Three of these are of manganin, two of which have the same length but different diameters, while two others have the same diameter but different lengths. The fourth wire is of German Silver. The resistance of each is to be determined.

Having connected one of the unknown resistances to the blocks X and G, find the approximate resistance of X by making the ratio arms equal (say 100 ohms each) and varying R until the galvanometer deflection reaches its minimum value. Note that if R be started at a small value and gradually increased, the galvanometer deflections will decrease to zero when the bridge is balanced. If R be still further increased the galvanometer will be deflected in the opposite direction from what it was at first. De-

flections of the galvanometer in the first direction then indicate that R is too small for balance, whereas deflections in the opposite direction correspond to values of R too large for balance. These relations should be borne in mind throughout the experiment. On account of the fixed resistance of the coils in R it may not be possible to obtain perfect balance (zero deflection), but when the deflection of the galvanometer is a minimum, R is approximately equal to X. Now find the approximate ratio between the approximate value of X and the total available resistance of R and adjust the ratio arms, R_1 and R_2, to *approximately* the same ratio. Vary R until balance is obtained (minimum deflection of galvanometer), when both keys are closed. Note the values of R, R_1 and R_2, and calculate X. In the same way determine each of the unknown resistances.

Note the length, diameter and material of each wire and calculate the resistance per centimeter, and the area of cross section.

To find the relation between the resistance of a wire of any given material and one of its dimensions, all of the other dimensions must be kept constant. Keeping this in mind show with your data that the resistance varies directly as the length and inversely as the cross section of the conductor by calculating the ratios of X and L for wires of the same material and diameter, and the products of X and A for wires of the same material and length. Also calculate the specific resistance of manganin and of German silver.

Questions.—1.—In Electrical Engineering the expression "resistance per mil-foot" is often used, meaning the resistance of a conductor one foot long having a cross sectional area equal to that of a circle whose diameter is one mil (one thousandth inch). Find the ratio between specific resistance and resistance per mil-foot of a substance.

2.—If a cylindrical conductor were stretched to $a(=2)$ times its original length, assuming its volume to remain constant, how would its resistance compare with its original resistance?

3.—An all metal telephone line (two wires) has a resistance of $a(=2.40)$ ohms when measured at the station, indicating that it is short circuited somewhere. If the wire is of copper with cross section $b(=0.1)$ sq. cm. and specific resistance 1.5×10^{-6} ohms per c. c. what is the distance of the short circuit from the station?

4.—In measuring an unknown resistance with a P. O. box bridge, the ratio arms were set at $a(=1000)$ and $b(=10)$ ohms respectively, and balance was obtained when the adjustable resistance (connected to the smaller ratio arm) was $c(=750)$ ohms. The P.D. of the cell was $d(=1.4)$ volts. Compute *a*) the unknown resistance, *b*) the current through it, *c*) the P. D. across it, and *d*) the current through the cell.

38. THE POTENTIOMETER.

To determine the electromotive force of several galvanic cells, singly and in combination, by means of the potentiometer. (D. 441, 454, 455 469-476; G. 812-831, 844-849, 925, 927; K. 622-642; W. 480, 490, 550, 557.)

APPARATUS.—Pye potentiometer with two storage cells, D'Arsonval galvanometer, dry cell, salammoniac cell, Daniel cell, two gravity cells and eight connecting wires.

DESCRIPTION AND THEORY.—A galvanic cell consists in general of two plates of different metals (or a plate each of a metal and some other substance such as carbon) placed a short distance apart in an acid or a solution of a salt, such that a difference of potential exists between the plates, and a current of electricity will flow from one to the other when

their external ends are joined by means of a conductor. *The electromotive force of such a cell is equal to the difference of potential existing between the external ends of these plates when they are not connected,* (*i. e.* there is an open circuit) or practically when they are joined by a very high resistance.

The electromotive force of cells may be compared by the potentiometer method. The simple potentiometer consists of a long homogeneous wire *AD* (Fig. 52) of uniform cross section. A constant fall of potential is produced in this wire by means of a source of constant current (in this case a storage battery *S. B.*) By Ohm's law the potential drop through a wire is equal to the current through it times its resistance. Since our wire is uniform and homogeneous, its resistance per unit length is the same throughout its length, and since the electromotive force (*e.m.f.*) of the storage cells is practically constant, the current strength is constant. It follows that the potential drop per unit length of wire is the same at all points.

A galvanic cell *B* is now connected in series with a sensitive galvanometer *G* to one end *A* of the wire and by means of a sliding contact key *S* to another point on the wire. This second cell also tends to produce a constant potential drop through *AS*. But *AS* is in parallel (See Exp. 36) with both of the circuits containing cells, and if like terminals (either + or −) of both cells are connected to the end *A* their *e.m.f.'s* will be opposed. The current from cell *B* will tend to divide at *A* part going through *AS* and part through the circuit *A*(*SB*)*DS*. The current from *S.B.* will tend to divide at the same point, part going through *AS* and part through the circuit *ABGKS*. Each branch current will be opposed by the electromotive force of the other cell and if the drop of potential through *AS* is not the same for both cells, a current will flow through it in one direction or the

other, depending upon which cell produces the greater drop, and there will be in general a deflection of the galvanometer. If now the sliding contact key S be moved along the wire a point may be found where there is no deflection of the

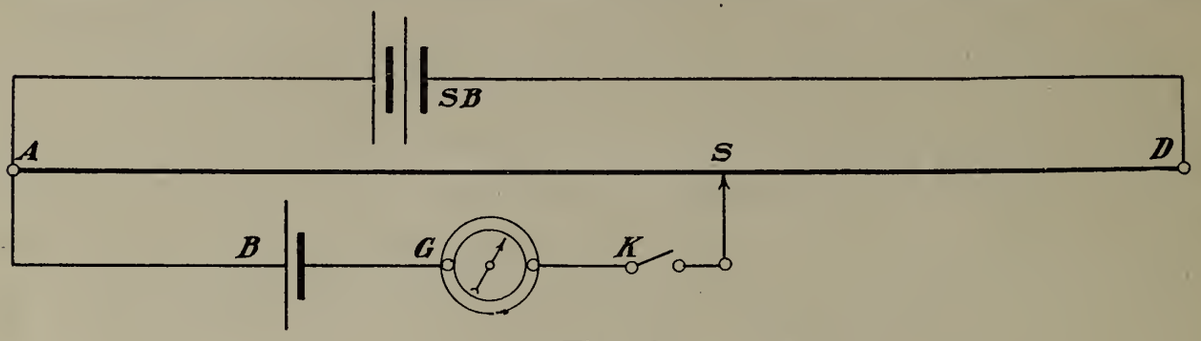


Fig. 52

galvanometer when the key is closed, because there is no current flowing in the galvanometer circuit. This indicates that the potential drop between A and S due to cell B is exactly equal to that due to cells $S.B.$, or in other words the tendency for cell B to send current through the galvanometer circuit from S to A is exactly neutralized by the tendency of cells $S.B.$ to send current through the same circuit. Since there is no current through the galvanometer circuit there will be no potential drop between the cell B and the points A and S so that the point A must be at the same potential as the positive terminal of B and the point S at the same potential as the negative terminal. Consequently the *e.m.f.* of the cell B must be equal to the potential drop in the section of wire AS due to the cells $S.B.$, and is therefore directly proportional to the length of wire AS.

A second cell C of known *e. m. f.*, such as a standard or a Daniel cell, may now be inserted in place of B and the contact key moved along the wire to some point S' where there will be no deflection of the galvanometer when the key is closed. The *e.m.f.* of cell C is then directly proportional to the length of wire AS'.

If E be the *e.m.f.* of a cell, L the length and R the resistance of the stretch of wire across which it is balanced, P. D. the potential drop through it, and I the current flowing in it, each with their proper subscript, we may write

$$\frac{E_B}{E_C} = \frac{P.D._B}{P.D._C} = \frac{IR_B}{IR_C} = \frac{L_B}{L_C}$$

Therefore

$$E_B = E_C \frac{L_B}{L_C}$$

The Pye potentiometer has a uniform high resistance wire, ten meters long, wound back and forth for convenience sake, in ten lengths of one meter each, on a board supplied with a millimeter scale. (See Fig. 53). The scale is calibrated from both ends so that lengths of wire from either end of a loop may be measured directly. The key S, is carried by a sliding triangular bridge, so that contact may be made at any point on any one of the ten lengths of wire. The storage batteries are connected to terminals A and D of the potentiometer wire, the primary cell circuit to A and to E or F, like poles of both being connected to A. Terminals E and F are at the ends of a metal strip on which one leg of the bridge rests, so that the current may pass from E or F to S through the bridge. The other two legs of the bridge rest on the wooden base and are therefore insulated. Care must be taken that the bridge does not come in contact with the binding post A, as this will cause a shortcircuit. When the potentiometer is balanced, the length of wire from binding post A to key S may be read on the attached scale.

The Daniel cell has a sheet of copper bent into cylindrical shape and immersed in a copper sulphate solution. A small pocket on the plate contains crystals of blue vitriol

for keeping up the concentration of the solution. Within this cylinder is placed a porous cup containing a zinc rod in dilute sulphuric acid. When properly set up this cell has an *e.m.f.* of 1.08 volts, which with proper care remains constant for a long time. The Daniel cell may therefore be used as a standard for comparison.

The gravity cell has the same elements as the Daniel cell, without the porous cup, depending upon the difference in the specific gravities of the copper sulphate and the zinc sulphate formed to keep the liquids apart.

The salammoniac cell has carbon and zinc plates in a solution of ammonium chloride, and the dry cell is practically the same except that the carbon plate is packed in some material that absorbs the solution and prevents polarization, and is contained in a zinc cylinder.

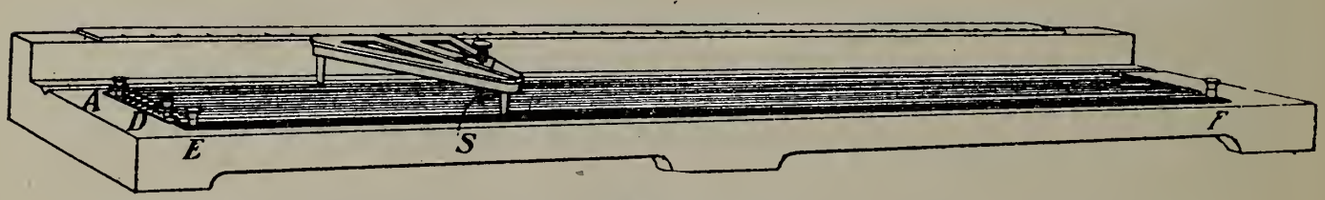


Fig. 53

The storage cells, in their common form, consist of two lead grids having their interstices filled with lead sulphate formed by making a paste of red lead and dilute sulphuric acid. These plates are immersed in a weak solution of sulphuric acid and are charged by passing a current from one to the other. On connecting the two together externally a current will flow in the opposite direction to that of the charging current. They are charged and discharged in this way a number of times to form them.

The Daniel and gravity cells, and the storage cells may be left in a closed circuit for some time without their *e.m.f.* depreciating markedly, *i. e.* they are closed circuit cells and

do not polarize easily. The salammoniac and dry cells are however open circuit cells, and can be used only for short intervals at a time, because their *e.m.f.* drops when they are in use due to polarization, and only recovers when they have been left on open circuit for some time.

In order that we may have the proper current strength or electromotive force in a given circuit, it is sometimes necessary to connect several cells in series, or in parallel, or in combinations of the two. (See Fig. 54.) To connect them in series the positive terminal of one is joined to the negative terminal of the next, etc., leaving one free positive and one free negative terminal to be connected to the given circuit. The potential difference from one end of the series to the other is evidently the sum of the potential drops through each cell, so that *the combined e.m.f. of the cells in series is the sum of their separate e.m.f's.* But it must be remembered that the effective (internal) resistance of such a cell is that of the liquid through which the current passes from one plate to the other (the resistance of the plates

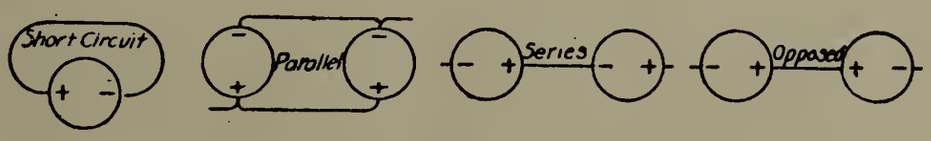


Fig. 54

themselves being negligible), and when several cells are in series the current must pass through their respective liquids successively, or *the total resistance of the cells in series is the sum of their separate resistances.* The current strength may be determined by Ohm's law, the total resistance including the internal resistance of the cell and the external resistance of the circuit, so that on short circuit (zero external resistance) the current strength is the same as that in a single cell. When several cells are connected in parallel, all the positive terminals are joined together and to one end of

the circuit, and all the negative terminals together and to the other end of the circuit. The potential difference from one set of terminals to the other is evidently the same as the potential drop through a single cell if they are all of the *same* kind, and *the combined e.m.f. of the cells in parallel is the e.m.f. of a single cell.* But the current passes through all of them simultaneously and though the length of the liquid path is the same for all of them as for one cell, the cross section for n of them in paralled is n times as large as that for a single cell, and since the resistance of a conductor varies inversely as its cross section, the resistance in this case will be $1/n$ times that of a single cell. As a result the current would be n times as great as from a single cell on short circuit

It is sometimes advantageous to combine both methods of connecting cells. For instance if we have 6 cells, we may connect them in 2 rows, each row having 3 cells in series. The 2 rows may then be joined in parallel by connecting their free positive terminals together at one end of the circuit and their free negative terminals together at the other end. Or we may connect them in 3 rows each having 2 cells in series and then join these 3 rows in parallel.

It may be shown that in order to obtain the maximum current from a combination of cells in a given circuit, they should be arranged in such a way that the internal resistance of the cells is equal to the external resistance of the circuit.

In the general case let n be the number of cells of the same kind to be connected, s the number in series in each row, p the number of rows joined in parallel, r the resistance and e the *e.m.f.* of each cell, and R the external resistance. Then the total number of cells will be $n = ps$. The *e.m.f.* of each row will be se and its resistance sr. The *e.m.f.* of p rows in parallel will be se also, but their combined

resistance will be sr/p. The current in the external circuit is therefore

$$I = \frac{se}{R+sr/p} = \frac{pse}{pR+sr} = \frac{ne}{pR+sr}$$

This current will evidently be a maximum when the denominator $pR+sr$ is a minimum. Suppose pR and sr to be the sides of a rectangle, their product being its area. This product $psRr$ is a constant in any given case, for the number of cells (ps) is fixed, the external resistance is fixed, and for a given current the internal resistance of the cells will be constant. But it is known that of all rectangles of a constant area that one has the least perimeter which has its sides equal, (the square.) Therefore the sum of two sides $pR+sr$ will be a minimum when $pR=sr$, or when $R=sr/p$. In other words, *in a series-parallel combination of cells, the current will have its maximum value when the external and total internal resistance are equal.*

When two cells have two like terminals joined together, and two to the external circuit, their *e.m.f.'s* are opposed to each other and their combined *e.m.f.* is the difference of their separate *e.m.f.'s*.

Directions.—Connect the apparatus as shown in the diagram, being careful to have the cells to be tested *opposed* to the storage cells, and the latter in series with each other. The Daniel cell to be used as a standard is to be tested first. By means of the sliding key, contact should be made with one wire after the other until two adjacent wires are found for which the galvanometer gives deflections in opposite directions. The key is then moved along between the contact points on these two wires and contact made at different points until the position s' is found for which there is no gal-

vanometer deflection. The length of wire AS' should be noted. The key must not be held in contact with the wire while it is being moved, and should not be pressed hard at any time, as this may injure the wire. Each of the gravity cells is then to be put in place of the standard and the balance positions found, the lengths AS being noted in each case. This is repeated with both gravity cells in parallel, then in series, and then in opposition. In the latter case the cell having the higher *e.m.f.* should be connected so that it has the same terminals (+ or −) joined to A and D as the storage cells have. Finally the dry cell and the salammoniac cell are each connected to the potentiometer and balanced. From the data obtained the *e.m.f.* of each cell and of each combination is to be determined by comparison with that of the Daniel cell. The *e.m.f.* of each combination should then be compared with the separate *e.m.f.'s* of the two gravity cells.

QUESTIONS.—1.—What *theoretical* arrangement of $a(=20)$ cells, each having an *e.m.f.* of $b(=1.9)$ volt, and an internal resistance of $c(=0.2)$ ohm, will send the maximum current through an external resistance of $d(=0.1975)$ ohm? Find the theoretical maximum current strength.

2.—Find the current strength through the above external resistance from these cells when arranged in all possible series-parallel combinations; that is, in parallel rows, each row having 1, 2, 4, 5, 10 or 20 cells in series respectively. What is the actual maximum current obtainable?

3.—Plot a curve with the data obtained in question, 2 having the number of cells in series as abscissae and the corresponding current strength as ordinates.

4.—A certain galvanic cell will send a current of $a(=0.28)$ ampere through an external resistance of $b(=3.)$ ohms.

When this resistance is changed to $c(=8.)$ohms the current drops to $d(=0.14)$ amperes. Calculate the *e.m.f.* and internal resistance of the cell.

39. FIGURE OF MERIT OF A GALVANOMETER.

To determine the figure of merit of a D'Arsonval galvanometer. (D. 437, 438, 452; G. 854; K. 602-605, 645; W. 477, 480, 486.)

Apparatus.—Resistance box (10,000 ohms) traveling plug, reversing switch, D'Arsonval galvanometer, galvanic cell, and connecting wires.

Theory and Method.—*The figure of merit of a galvanometer is defined as the strength of that current which on passing through the coil of the galvanometer produces a deflection of one scale division.* If the deflections are observed by the telescope and scale method, the figure of merit is usually calculated for a deflection of one millimeter with a distance of one meter between scale and mirror. The method is an application of Ohm's law. The apparatus is arranged as shown in the diagram, Fig. 55. A gravity cell of known *e.m.f.* is connected in series with the D'Arsonval galvanometer G of known resistance, and a variable resistance R through a reversing switch. The galvanometer coil of fine

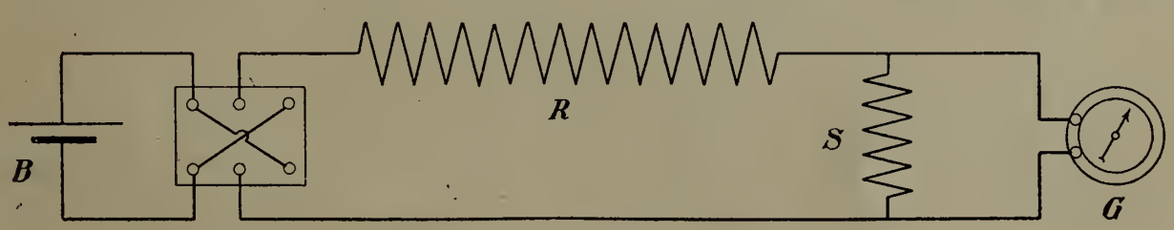


Fig. 55

wire cannot safely stand a current of more than about 0.00001 ampere, and with an *e.m.f.* of one volt in the circuit, R would have to be very large to reduce the current strength to this amount. On the other hand if the current in the gal-

vanometer were reduced by using a shunt, (the resistance R being omitted) the shunt resistance would have to be inconveniently small. A combination of both series and shunt resistances is therefore used, as shown in the diagram.

Let S be the resistance of the shunt, G that of the galvanometer, B that of the cell, and E the *e.m.f.* of the latter. The total resistance of the circuit is then $R+B+(SG/(S+G))$ and the current strength in the main circuit is by Ohm's law,

$$I=\frac{E}{R+B+\frac{SG}{S+G}}$$

The resistance of the cell and that of the galvanometer and shunt in parallel are so small compared with the values of R to be used, that they may be neglected in calculating I. But the current in the galvanometer is

$$I_g=I\frac{S}{S+G}$$

(See Exp. 36) and if d be the deflection in millimeters corresponding to this current strength, the figure of merit will be

$$F=\frac{I_g}{d}=\frac{IS}{d(S+G)}=\frac{ES}{Rd(S+G)}$$

A galvanometer may be calibrated to be used either as an ammeter, an instrument for measuring current strength in amperes, or as a voltmeter, an instrument for measuring the drop in potential through a circuit, in volts.

AMMETERS.—As an ammeter it must be connected in series with the apparatus through which the current is flowing and should not appreciably alter the total resistance of the circuit, as this would change the current to be measured.

Since the galvanometer usually has considerable resistance, it would appreciably reduce the current strength in the circuit when it is introduced. On the other hand the galvanometer must be protected from heavy currents by means of a shunts, and as their combined resistance $SG/(S+G)$ is less than that of either S or G alone, we may, by using a suitably small shunt, make the combined resistance as small as we please. Hence the change in current strength when the galvanometer and shunt are inserted in the circuit, may be made negligible. The galvanometer and shunt resistance may be so chosen for a given circuit, that the deflections per unit current will have any value we please, say one centimeter per ampere of current in the main circuit.

Voltmeters.—When used to measure the drop in potential through any part of a circuit, the galvanometer must be placed in parallel with it, so that there is the same drop of potential through both. At the same time the current strength through the circuit should not be appreciably altered, for this would alter the drop of potential which is to be measured, the two being directly proportional, according to Ohm's law.

If R be the resistance of the circuit across which the potential difference is to be measured, and G that of the galvanometer, their combined resistance in parallel is $RG/(R+G)$. The greater the value of G the smaller will be the difference between R and $RG/(R+G)$, for $G/(R+G)$ will be nearly unity. If then, the resistance of the galvanometer circuit be very large, its insertion in parallel with the main circuit will not appreciably alter the total resistance of the latter, nor the current strength, hence the drop of potential through the circuit will be practically the same before and after inserting the galvanometer. If the galvanometer itself has not a high resistance, a high re-

sistance may be placed in series with it. The current strength through the galvanometer (and therefore its deflections also) will be directly proportional to the potential drop which is being measured. A high resistance galvanometer is usually one with a great many turns of fine wire in its coils, and is therefore quite sensitive to small currents.

DIRECTIONS.—Connect the apparatus as shown in the diagram, Fig. 55. Instead of using two separate resistance boxes for R and S use a single box with a traveling plug (a plug furnished with a binding post) in place of the 10 ohm plug. (See Fig. 57 for a similar arrangement. Here the traveling plug is shown at D.) Connect the low resistance coils (1 to 4 ohms) as a shunt to the galvanometer, the high resistance coils in series with it. Vary the shunt resistance from one to four ohms and the series resistance from 4000 to 10000 ohms as shown on the data sheet, and observe the deflections in millimeters for each case with the current direct and reversed. With the mean deflection in each case calculate the figure of merit for a scale distance of one meter, remembering that the scale is actually only a half meter's distance from the galvanometer coil.

QUESTIONS.—1.—Assuming the internal resistance of the cell to be $a(=2)$ ohms, what is the % error in your value of the figure of merit due to neglecting $B+SG/(S+G)$ in your calculation, when the series resistance is $c(=4000)$ ohms?

2.—A galvanometer whose figure of merit (as defined) is $a(=10^{-7})$ is to be used as an ammeter. If its resistance is $b(=400)$ ohms, calculate the necessary shunt resistance to make it read $c(=1.)$ m. m. deflection at a meter distance per $d(=0.001)$ ampere in the main circuit.

3.—A galvanometer whose resistance is $a(=500)$ ohms, has a shunt of $b(=1.)$ ohm, and a figure of merit of $c(=10^{-8})$.

What series resistance must be used with this apparatus so that it may serve as a voltmeter, reading $d(=1.)$ volt per $e(=1.)$ cm. deflection at a half meter distance.

4.—A galvanometer with figure of merit $a(=2\times10^{-7})$ and a resistance $b(=2200)$ ohms is placed in parallel with a circuit having a resistance $c(=5)$ ohms. If the galvanometer reading is $d(=10)$ cm. on a scale at $e(=50)$ cm. distance, what is the current strength in the main circuit?

40. RESISTANCE OF A GALVANIC-CELL BY OHM'S METHOD

To determine the internal resistance of a cell, or the ratio of the *e.m.f.* of the cell to the strength of the current which passes from one plate to the other, through the liquid of the cell. (D. 442, 443, 469-476; C. 837, 845, 846, 927, 944; K. 622, 623, 625; W. 478, 480.)

Apparatus.—Gravity cell, tangent galvanometer, resistance box, reversing switch, and connecting wires.

Theory and Method.—The method used is a simple application of Ohm's law to a circuit containing the cell whose resistance is to be measured in series with a tangent galvanometer and a resistance box. A switch for reversing the current in the galvanometer is also included in the circuit.

Let E be the *e.m.f.* of the cell, B its internal resistance, G that of the galvanometer, D its deflection, and K its reduction factor. Then if the box resistance be R' the current strength will be

$$I' = K\ tand' = E/\ (R' + G + B)$$

If the box resistance is now changed to a value R'' the corresponding current will be

$$I'' = K\ tan\ d'' = E/\ (R'' + G + B)$$

from these two equations we may eliminate K and E and obtain an expression for B in terms of known quantities.

$$B = \frac{tand''\ (R''+G) - tand'\ (R'+G)}{tand' - tand''}$$

DIRECTIONS.—Connect the apparatus as shown in the diagram, Fig. 47. Vary the resistance R until the deflection is about 30°, and note the deflection of both ends of the pointer with current direct and reversed. Change the resistance R until the deflection is about 35° and note the deflections as before. Repeat this twice more with angles of 40 and 45°, and 55 and 60° respectively, and calculate B using the mean deflections.

QUESTIONS.—1.—Explain in detail why this experiment is adaptable to cells which do not polarize rapidly, and to no others?

2.—Supposing a tangent galvanometer were not available, derive the expression for the internal resistance of a cell when using a shunted D'Arsonval galvanometer as the ammeter. (See Exp. 39.)

3.—A battery is in series with a resistance R and a D'Arsonval galvanometer of resistance $a(=200)$ ohms which is shunted by a coil of $b(=2)$ ohms resistance. When R is $c(=5000)$ ohms the deflection is $d(=10$ cm.), and when R is $e(=9000)$ ohms, the deflection is $f(=5.6)$ cm. Find the average resistance of the battery.

41. POTENTIAL DIFFERENCE AT THE TERMINALS OF A CELL AS A FUNCTION OF THE EXTERNAL RESISTANCE

To show that the potential difference at the terminals of a galvanic cell depends upon the external resistance, and that for a high value of this resistance the potential differ-

ence approaches a fixed value, the *e.m.f.* of the cell. Also to determine the *e.m.f.* and the internal resistance of the cell, by a graphical method. (D 442, 443, 452; G. 844, 845, 854; K. 641, 642; W. 486, 490.)

Apparatus.—D'Arsonval galvanometer and shunt, gravity cell, resistance box with traveling plug, reversing switch, eight connecting wires.

Theory and Method.—The *e.m.f.* of a cell was defined as *the difference of potential between its terminals on open circuit*, that is when no current is flowing through it. If now the circuit external to the cell be closed through a resistance which is not infinite, a current will flow through the circuit, and the potential difference at the terminals of the cell will no longer equal its *e.m.f.* The current strength in the circuit will equal the total electromotive force in the circuit divided by the sum of all the resistances, and in this case the circuit is completed through the internal resistance of the cell. Hence if E be the *e.m.f.* of the cell, B its internal resistance, R the external resistance, and I the current, we have by Ohm's law

$$I = E/(R+B)$$

or

$$E = IR + IB \qquad (1)$$

The difference of potential *P.D.*, between the terminals which equals the drop of potential IR through R, is less than E by the drop of potential $e(=IB)$ through the internal resistance B, or

$$P.D. = E - e$$

It is evident that as R is increased, I (and therefore e also) will become smaller and smaller so that the value of *P.D.*

will gradually approach that of E. *When R is infinitely great*, or practically when there is an open circuit,

$$P.D. = E$$

For equal values of external and internal resistance the drop of potential through the two circuits is the same, and equation (1) becomes $E = 2IR$, therefore

$$P.D. = e = E/2$$

Bearing these facts in mind and plotting a curve with values of R as abscissae and corresponding values of $P.D.$ as ordinates, we may readily determine E as the limiting value of $P.D.$ as R becomes infinite, (*i. e.* as the ordinate of the asymptote to the curve.) From the same curve, B may be determined by measuring the abscissa of that point on the curve for which $P.D. = E/2$ (*i. e.* that point which lies half way between the asymptote and the axis of abscissae.)

If we plot reciprocals of $P.D.$ and of R as coordinates the curve will be a straight line. The relation between $P.D.$ and R may be found from equation (1). The ratio of E to $P.D.$ is evidently

$$E/P.D. = I(R+B)/IR$$

or,

$$\frac{E}{P.D} = 1 + \frac{B}{R} \qquad (2)$$

If we put $1/R$ equal to zero in equation (2), then $E/P.D. = 1$, or

$$E = P.D.$$

But the point on the curve whose abscissae is $1/R = 0$, has for its ordinate $1/P.D.$ the portion of the axis of ordinates intercepted by the curve. Therefore *the e.m.f. of the cell will equal the value of P.D. whose reciprocal is represented by the intercept of the curve on the axis of ordinates.*

If we put $1/P.D.$ equal to zero in equation (2), then $1+B/R=0$, or

$$B=-R$$

But the abscissa $1/R$ corresponding to $1/P.D.=0$ is the intercept of the curve on the axis of abscissae, and since it lies to the left of the origin it will have a negative sign, so that $-R$ will be positive. Therefore *the internal resistance of the cell will equal the value of R whose reciprocal is represented by the intercept of the curve on the axis of abscissae.*

In order to measure the drop of potential $P.D.$ through R, a shunted galvanometer with a high resistance R_1 in series as shown in figure 56, is used as a voltmeter. (See Exp. 39). The potential difference between the terminals C and D or between the terminals of the cell B will equal the drop of potential through the galvanometer circuit, or its resistance times the strength of the main current in the galvanometer

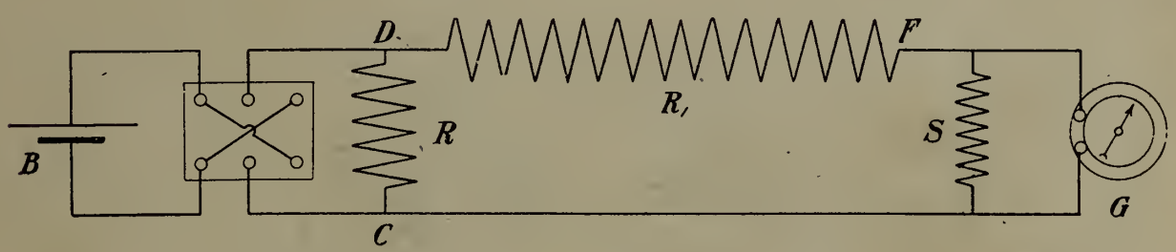


Fig. 56

circuit. If I_g is the galvanometer current, the current in R_1 will be $I_g\ (S+G)/S$. The total resistance will be $R_1 + (SG/(S+G))$, but if R_1 is large, say 10,000 ohms, $SG/\ (S+G)$ will be negligible in comparison. Then the drop of potential through CD will be

$$V=R_1 I_g\ ((S+G)/S)$$

If I_g be taken as the galvanometer current in amperes necessary to produce a deflection of one mm. (the figure of merit), V will be the potential difference in volts between

C and D for one mm. deflection. For a deflection of d mm., the number of volts will be

$$P.D. = d \times V. = dR_1 I_g \frac{(S+G)}{S}$$

DIRECTIONS.—The apparatus is arranged as shown in figure 56. The galvanometer has a low resistance shunt of about 4 ohms and a resistance of 10,000 ohms in series. The series resistance R_1 and the external resistance R are included in the same box, connection being made to a traveling plug at D. This is shown in figure 57, where the box terminals are marked F and C to correspond with figures 56, and the traveling plug is marked D. The figure shows the plugs removed to place 10000 ohms in series with the galvanometer. Vary the resistance CD from infinity to one ohm as shown on the data sheet and note the corresponding

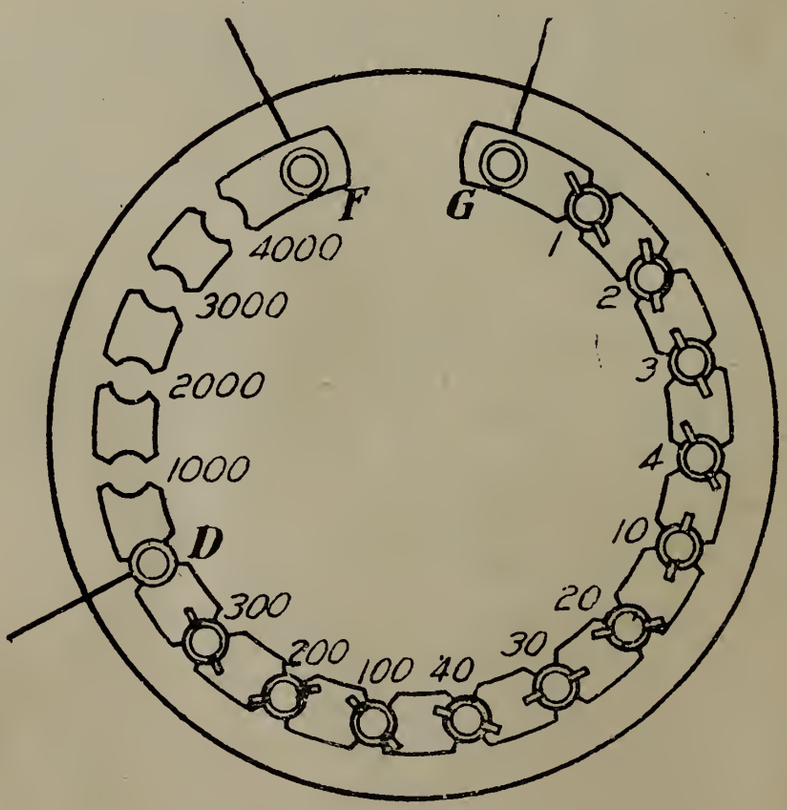


Fig. 57

deflections of the galvanometer. To make R infinitely large, disconnect the two wires leading from the terminal C of the resistance box to the galvanometer and switch, but keep these wires joined together. From the value of the figure of merit of the galvanometer, the values of $P.D.$ for different values of R are to be calculated. The two curves mentioned above are to be plotted and the value of E and of B should be obtained from each of them.

Questions.—1.—Plot the two curves described in the above experiment. What are the *e.m.f.* and internal resistance of the cell as determined from each curve?

2.—If you used a galvanometer, whose resistance is $a(=250.)$ ohms, with a shunt of $b(=4.)$ ohms and a series resistance of $c(=4000.)$ ohms, what is the % error in your calculated value of $P.D.$ due to neglecting $SG/(S+G)$?

3.—A shunt wound dynamo has its field coils connected between the terminals, in parallel with the external circuit. If the armature (internal) resistance of such a dynamo is $a(=0.0404)$ ohm and its field resistance is $b(=3.75)$ ohms, what must be the $P.D.$ at the terminals to properly light $c(=200)$ incandescent lamps in parallel, each lamp requiring $d(=0.5)$ ampere at $e(=100)$ volts, the resistance of the connecting leads being $f(=0.1)$ ohm? What *e.m.f.* must be generated in the armature under these conditions?

4.—A certain battery with $a(=4.5)$ volts *e.m.f.* will send $b(=1.5)$ amperes through an external resistance of $c(=1.)$ ohm. Through what resistance will it send $d(=0.01)$ ampere?

42. ELECTROLYSIS AND THE COPPER VOLTAMETER

To measure a current strength with the copper voltameter, and by passing this current through a tangent galvanometer to determine the horizontal intensity of the earth's field from its reduction factor. (D. 462-468; G. 939-941; 953-956; K. 608-620; W. 539, 540, 478.)

APPARATUS.—Voltameter jar with copper sulphate solution, two copper anodes, one copper kathode, electrode holder, tangent galvanometer, reversing switch, fine balance and weights, fine sand paper, filter paper, connecting wires, and 110 volt circuit with lamp resistance.

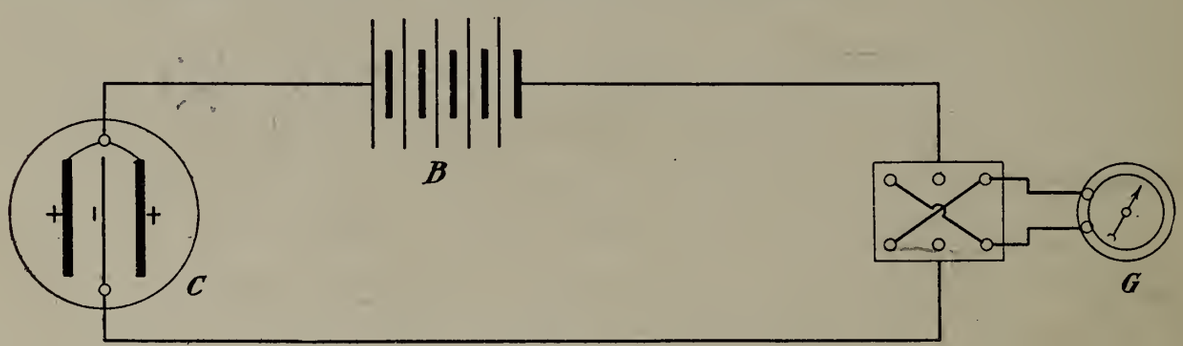


Fig. 58

THEORY AND DEFINITIONS.—A solution through which a current of electricity is conducted by a process similar to convection, is termed an electrolyte, and the process is called electrolysis. The dissolved substance is dissociated by the act of solution; that is, some of its molecules separate into two parts called ions and the weaker the solution the greater is the number of free ions. *These ions are the constituent atoms or groups of atoms, one bearing a positive charge of electricity, the other a negative charge.* A current of electricity may be passed through the solution between two conductors dipping into it. These terminal conductors are called *electrodes*, and *that one by which the current enters the solution is known as the anode, the one by which the current*

leaves it, the kathode. The electrolyte and the electrodes form an electrolytic cell.

When a current of electricity passes through an electrolytic cell *the positively charged ions move in the direction of the current, toward the kathode.* They are therefore called *kations. The negatively charged ions move toward the anode and are called anions.* These anions and kations alternately separate and combine with each other, and in this process some of them will appear at the electrodes, where they generally give up their charge. They may adhere to the electrode as in the case of most metals, or may escape from the solution, as in the case of gases of less density than the latter, or they may enter into secondary relations with the surrounding solution.

The laws of electrolysis, known as Faraday's laws, may be stated as follows:—*The mass of the ions liberated at either electrode is proportional 1st. to the quantity of electricity which passes through the solution. 2nd. to the chemical equivalent of the ion.*

The quantity of electricity in coulombs equals the current strength I in amperes multiplied by the time t in seconds during which the current flows. The *chemical* equivalent of the ion is its atomic weight w divided by its valency v in the electrolyte used. Hence the mass m of ions liberated is

$$m = k\frac{w}{v}It = zIt$$

The quantity $z = kw/v$ is a constant for any one electrolyte and is called the *electrochemical* equivalent of the ion. If we put It equal to unity we see that m equals kw/v, or *the electrochemical equivalent may be defined as the mass of the ion set free by unit quantity of electricity.*

Taking the atomic weight and valency of hydrogen both

equal unity, its electro chemical equivalent is numerically equal to k. That is to say, the proportionality factor k is numerically equal to the mass of hydrogen ions in grams liberated by one coulomb of electricity. But when the atomic weight of oxygen is used as the base of the system of weights, this relation no longer holds true for the chemical equivalent of hydrogen instead of being unity will be 1.008.

The value of k may be determined by electrolysis of a solution in which hydrogen is liberated, but is more accurately obtained from a determination of the electrochemical equivalent of silver from a silver nitrate solution. The value of k is found to be $1.044 \times (10^{-5})$ grams per coulomb.

Knowing the electrochemical equivalent of an ion, we may measure the mass of it liberated in a given time and so determine the current strength which liberates it. An electrolytic cell used for this purpose is called a coulombmeter or voltameter. The copper voltameter may consist of two copper plates in a solution of copper sulphate. (Density 1.15 to 1.18. 20 to 25 grams of copper sulphate crystals in 80 grams of water, with 1% of concentrated sulphuric acid added). The solution dissociates into copper kations and sulphion (SO_4) anions. The copper anode goes into solution by combining with the anions liberated in its neighborhood, so that the solution surrounding it becomes more concentrated. The ions liberated at the kathode are deposited upon the latter, the solution in its neighborhood becoming less concentrated. Taking the atomic weight of copper as 63.64 and its valency as 2, its electrochemical equivalent will be 0.0003294 grams per ampere second. The gain in mass of the kathode in a given time may be accurately determined, and from it we can readily calculate the current strength.

Directions.—Sandpaper the lugs on the electrodes and the clips on the holder so that they make good contact with each other, and sandpaper the electrodes so that they present a fresh, unoxidized surface toward each other. Clamp the lugs in the holder, the thin kathode being symmetrically placed between the two heavier anodes, so that the plates are parallel to and level with each other. Place them in the jar and fill it with copper sulphate solution up to the lugs on the plates. Connect the apparatus as shown in figure 58, so that the current from the source B passes through the electrolytic cell C in series with the tangent galvanometer G, and can be reversed, in the galvanometer only, by the reversing switch. Connect to the smallest coil of the galvanometer (middle terminals) so that a large current may be used. The current strength is governed by the lamp resistance. The positive terminal of the source of current must be connected to the anodes. The terminals of B should be marked + and −, but if necessary, the direction of the current may be determined by noting on which plate the current deposits copper in a preliminary trial, or by noting the deflection of a compass needle placed under the connecting wires.

Allow the current to flow for five minutes in order to give the kathode a preliminary coating. Then remove the kathode, wash it carefully by flowing soft water over it, and dry by pressing gently between sheets of filter paper without rubbing. The copper will not adhere to the plates unless they are perfectly free from oil or grease. Those parts of the plates which are to be immersed, must therefore not be touched by the hand nor laid on a dusty table or balance pan. When quite dry the kathode should be carefully weighed on the fine balance and the mass recorded. Replace the plates in the cell and close the circuit, noting the exact

time when the current starts to flow. Read the deflections of both ends of the galvanometer pointer and at the end of five minutes reverse the current. While throwing over the reversing switch the current will be interrupted. It is therefore necessary to reverse as rapidly as possible. Repeat the procedure, reversing the current and reading the galvanometer deflections at the end of every five minutes until the current has been flowing for one hour. Then open the switch, remove the kathode, and wash, dry and weigh it as before, and record the mass. The copper sulphate solution should be returned to the supply jar.

While the copper is being deposited, the number of turns of wire on the galvanometer coil, and the mean radius of a turn should be recorded, and from these the galvanometer constant G calculated. Determine the mean current strength from the change in mass of the kathode. As the copper plates dissolve in the electrolyte at a definite rate per second per unit area of electrode surface, it is usually necessary to allow for this in calculating the amount of copper deposited but for the small plates used and the short time of deposit, this correction is negligible. Note the tangent of the mean angle of deflection and calculate K, the reduction factor of the galvanometer. The horizontal component of the earth's field is calculated from the galvanometer constant and the reduction factor.

QUESTIONS.—1.—What is the advantage of the tangent galvanometer over the D'Arsonval for this experiment? (*b*)—Why is the switch reversed every five minutes?

2.—A current of $a(=1)$ ampere will decompose how much $b(=$ water) in $c(=1)$ hour if the atomic weight of $d(=$ hydrogen) is $e(=1.008)$?

3.—A voltameter containing a solution of a salt of $a(=$ nickel) with a resistance of $b(=\frac{1}{2})$ ohm between its ter-

minals, has a constant potential difference of $c(=2)$ volts maintained between them. How much of a will be deposited in $d(=15)$ minutes?

4.—A copper voltameter is in series with a tangent galvanometer. A current passing through it deposits a $(=0.912)$ grams of copper on the kathode in $b(=20)$ minutes, and the galvanometer deflection is $c(=30)°$. What is the reduction factor of the galvanometer?

43. ELECTROMAGNETIC INDUCTION

To study electromagnetic induction qualitatively (D. 499-505; G. 957-963; K. 710-726; W. 516-519.)

Apparatus.—A primary and secondary coil of wire, ballistic D'Arsonval galvanometer, reversing switch, horseshoe magnet, iron core, 110 volt circuit with lamp resistance.

Theory and Definitions.—Whenever the number of lines of force threading through a closed circuit is altered, a current of electricity is induced in the circuit. This phenomenon is called electromagnetic induction. Whenever the induction results from relative motion between the circuit and a source of magnetic lines of force, *Lenz's law states that the induced current is always in such a direction that it opposes by its electromagnetic action, the motion which produced it.*

The change in the number of lines of force may be due to

(*a*) Relative motion between the closed circuit and a magnet;

(*b*) Relative motion between the closed circuit and a second closed circuit through which a steady current is flowing;

(*c*) A change in the strength of the current flowing through the second closed circuit when both circuits are at rest.

In cases (*b*) and (*c*) that circuit in which the current is induced is called the secondary circuit, the other is the primary circuit. The current lasts only during the time that the change in the number of lines is taking place, and a ballistic galvanometer must be used to indicate the total flow of current.

It is sometimes more convenient to think of the current as induced in a conductor by cutting across lines of force. In this case the direction of the induced current is given by Fleming's rule. Place the thumb, forefinger and middle finger of the *right* hand mutually perpendicular to each other. Then *if the thumb be pointed in the direction of motion and the forefinger in the direction of the lines of force, the middle finger will point in the direction of the induced current.*

The permeability of iron is greater than that of air; that is, for the same field strength, there will be more lines of force in a given cross section of iron than in the same cross section of air at the same point in the field. Hence if we place an iron core through one or both of the circuits, the change in the number of lines of force in the secondary will be greater than before.

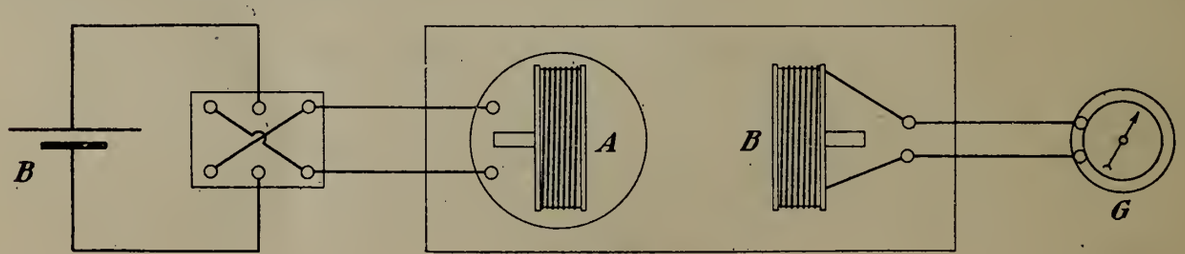


Fig. 59

DIRECTIONS.—Connect the apparatus as shown in the diagram, Fig. 59. The secondary coil *B* is fixed, and connected to the galvanometer *G*. The primary coil *A* is mov-

able and may be turned about a vertical axis or may be moved to different distances from *B*. The coil *A* is connected to the source of current *B* through the reversing switch *K*.

(1) *Relative motion between a magnet and a closed circuit.*—Bring the horse-shoe magnet rapidly astraddle of the coil *B* and note the direction of the deflection of the galvanometer. When the galvanometer coil has returned to its position of rest, rapidly withdraw the magnet from the coil and note the deflection. Repeat this with the magnet poles reversed.

Relative motion between two circuits.

(2) *Primary and secondary circuits approaching or receding from each other.*—Place the primary coil *A* parallel to the secondary coil at the opposite ends of the supporting board, and close the switch so that the current flows through *A*. Move *A* suddenly up to *B* and note the direction of deflection. When the galvanometer is again at rest, withdraw *A* and note the deflection. Reverse the current and repeat.

(3) *Rotation of primary circuit.*—Insert the brass pin through the holes in the base of the coil and the supporting board and place *A* parallel to *B*. Rotate *A* suddenly through 90° and note the amount and direction of deflection. When the galvanometer is again at rest, rotate *A* through a further 90° in the same direction and at approximately the same speed, and note the direction and amount of deflection. Compare the two deflections. Then bring *A* back to its original position and when the galvanometer is at rest, rotate it in the same direction as before and with approximately the same speed, but through 180°. Allow the galvanometer to come to rest and rotate *A* through another 180° as before. Compare the two deflections in this case in direction and amount.

(4) *Variable current in the primary.*—Bring *A* close up to and parallel to *B*, and when the galvanometer is at rest, suddenly close the primary circuit and note the deflection. After the galvanometer is again at rest, suddenly open the switch. Repeat with the switch reversed. Compare the four deflections.

(5) *Primary at different angles.*—Place *A* on the brass pin and note the deflections on breaking the circuit when the plane of coil *A* makes the following angles with the plane of *B*:—0°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 180°. Plot a curve having angles as abscissae and deflections as ordinates.

(6) *Effect of iron core.*—With *A* and *B* parallel and about 3 cm. apart, note the deflection when the current is broken. Put an iron core through the center of both coils and repeat. Compare the deflections.

QUESTIONS.—1.—With the data obtained in section five of the above experiment plot a curve having angles between the planes of primary and secondary coils as abscissae and corresponding deflections of the galvanometer as ordinates.

2.—Suppose a circular metal ring to rotate clockwise about an axis through its center, perpendicular to the paper but parallel to its own plane, in front of the north pole of a permanent magnet lying in the plane of the paper. Draw four diagrams showing the ring in its four quadrants, and indicate on each the direction of the induced current, and that of the lines of force which it sets up in the ring.

NOTE.—Draw a section through the center of the coil parallel to the plane of the paper, representing the cut ends of the coil by circles. A dot or dash in one of these circles represents an approaching current (point of an arrow); a cross represents a receding current (tail of the arrow.)

3.—Explain how Lenz's law applies in each case (by the aid of the north and south faces of the ring,) or how Fleming's rule applies.

4.—At a place where the vertical component of the earth's magnetic field is $a(=0.50)$, a horizontal coil of wire of $b(=10)$ turns, with diameter $c(=3)$ meters, and resistance $d(=2$)ohms, is suddenly pulled out into a straight line loop in $e(=\frac{1}{2})$ second. Find *a*) the induced *e.m.f.*; *b*) the average current in amperes; *c*) the total flow of electricity in coulombs; and *d*) the energy used up.

44. JOULE'S LAW AND THE ELECTRO-CALORIMETER

To determine Joule's equivalent by the electro-calorimeter. (D. 458-461; G. 403, 858-861; K 654-657; W. 493, 494.)

Apparatus.—Calorimeter with high resistance coil, tangent galvanometer, reversing switch, source of current, thermometer, coarse balance and weights, P. O. box bridge, dry cell, connecting wires and watch or clock.

Definitions and Theory.—Whenever a current of electricity flows through a conductor it generates heat. If the difference of potential between the terminals of the conductor be V the energy expended when unit quantity of electricity flows through it will be V units by definition of difference of potential. If in the time T, q units flow through the conductor, the work done will be

$$W = qV$$

But if i be the current strength, $q = iT$ and if r be the re-

sistance of the conductor, $V = ir$, by Ohm's law, whence

$$W = i^2 r T$$

When i, r and t are measured in *C. G. S.* absolute units, the work done will be expressed in ergs, but if C be the current in amperes and R the resistance in ohms, since one ampere equals 10^{-1} *C. G. S.* units of current and one ohm equals 10^9 *C. G. S.* units of resistance, the work done will be

$$W = 10^7 \ C^2 R T \text{ergs} = C^2 R T \text{ joules}$$

the joule being defined as a unit of energy equal to 10^7 ergs. Then if one ampere flows through a resistance of one ohm for one second the work done will be one joule. If we wish to express the energy expended as heat in calories we must divide by the mechanical equivalent of heat, J, in ergs per calorie, whence

$$H = \frac{10^7 \ C^2 \ R \ T}{J} \text{calories.}$$

This is known as Joule's law.

By passing a current of electricity through a coil of wire immersed in a water calorimeter and measuring the strength of current, resistance of coil, time of flow and heat generated, we may determine Joule's equivalent, J, by the above formula. If however, we assume the value of J, we may determine the resistance of the wire or the strength of the current.

DIRECTIONS.—Connect the coil of calorimeter and tangent galvanometer in series with the source of current and arrange the switch so that the current may be reversed. Before starting, measure the resistance of the coil and weigh the calorimeter and stirrer. Cool some water to about 10 degrees below the room temperature and fill the calorimeter

about three-fourths full. Determine the weight of water added. Note the temperature of water, and at the instant of closing the circuit note the exact time and observe the galvanometer deflection. At the end of every five minutes reverse the current and observe the deflection. Keep the water well stirred. Watch the thermometer and when the temperature has risen to about 10 degrees above that of the room, open the circuit, carefully observing the time and temperature at the instant of opening. Redetermine the resistance of the coil, and find the mean of these two determinations. Calculate the water equivalent of the calorimeter cup and stirrer. Determine the heat generated from the change in temperature. Average the galvanometer deflections and determine the current strength. With this data determine the value of J.

Questions.—1.—Given two conductors of the same material and length but of different cross sections. In which will the energy loss per unit time by heating be the greater *a*) when they are in series? *b*) When they are in parallel? Give reasons for your answers. (Notice that $C^2R = E^2/R$.)

2.—If coal costing $a(=4)$ dollars per ton develops $b(=8000)$ calories of heat per gram, and electricity costs $c(=10)$ cents per kilowatt hour, how does the cost of heating by coal compare with that of heating by electricity?

3.—An electric motor of $a(=2)$ kilowatt capacity, operating at $b(=100)$ volts, receives current through a pair of copper wires from a station which is $c(=1)$ kilometer away. If the energy loss per second due to heating in transmission may be only $d(=1)$ % of the full capacity of the motor, what must be the diameter of the wire?

4.—If the electricity in problem 3 were transmitted at $e(=2000)$ volts, and the same loss of energy by heat were allowed, how much would be saved on the cost of the transmission lines when copper is at $f(=15)$ cents per pound?

Experiments in Light.

PHOTOMETRY.

Definitions and Theory.—(D. 627-629, 712, 713; G. 519-521; K .793-800; W. 358-361.)

The comparison of intensities of illumination is called photometry and a photometer is a device for making such comparisons. It may be shown that the intensity of illumination, or amount of light per unit area, varies inversely as the square of the distance from the source. That is if I' be the intensity at a unit distance from the source, the intensity at a distance r will be $I = I'/(r^2)$. Photometers are usually so arranged that the intensity of illumination produced on a screen by the source to be tested, may be made equal to the intensity (on the same screen) from the standard with which it is to be compared. This may be done by keeping the two sources fixed in position on the photometric

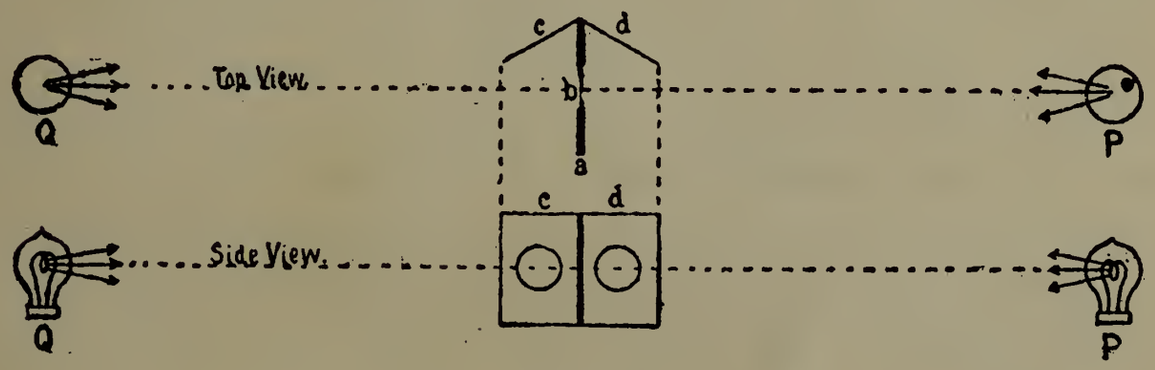


Fig. 60

bench and moving the screen on the bench between them, until the intensity of illumination is the same from both sources. See Fig. 60. One of the first standards to be used was a candle made according to special specifications and burning at a definite rate. The intensity of illumination is therefore often expressed in "candle-feet," that is, in terms

of the number of standard candles which would, at a distance of one foot from the center of the flame (in the horizontal plane through this point), give the same intensity as the source in question at the same distance. Suppose we are comparing two sources of illumination, P and Q, and that when the screen is equally illuminated by both, its distances from P and Q are p and q respectively. Let I' be the intensity of illumination one foot from P and I'' the intensity one foot from Q. Then the intensity at the screen will be

$$I = \frac{I'}{p^2} = \frac{I''}{q^2}, \text{ or } I' = \frac{P^2}{q^2} I''$$

I' and I'' may be termed the illuminating power of the two sources and if Q is a standard candle, I'' will be one candle power and I' will be expressed in terms of candle power. Various devices are used to determine just when the intensities from both sources are the same. In the Bunsen photometer a screen of white paper with a grease spot (b, Fig. 60) at the center has its two faces each illuminated by one of the two sources. This spot is translucent and therefore appears darker than the surrounding paper by reflected light and lighter by transmitted light. When the two faces are equally illuminated the grease spot will almost disappear, for then the loss in amount of light reflected from the spot on one side will be compensated by the light transmitted from the other side of the screen. In order to make both sides of the screen visible at the same time, two plane mirrors, c and d, are set at an angle with it. It is advisable to view both images in the mirrors with the same eye, for the two eyes are seldom able to judge equality of intensity with the same degree of accuracy.

If a transparent or translucent screen be interposed between the source and the photometer screen, the amount

of light which passes through will depend upon the thickness of the screen. Let α be the fractional part of the light which passes through unit thickness of the screen. Then if I be the intensity before passing through the screen, the intensity after passing through a screen of thickness of n units will be $I\alpha^n$. For consider the screen to be made up of layers of unit thickness. The intensity after passing through the first layer will be $I\ \alpha$, and only the fraction α of this amount, or $I\alpha^2$, will pass through the second layer: only the fraction α of this amount, or $I\alpha^3$, will pass through the third layer, etc. Let the intensity of the source P through a screen of thickness t, be I'_1 and let the distances from the photometer screen be p_1 and q_1 when the latter is equally illuminated by both sources. Then

$$I_1' = (p_1^2 \div q_1^2) I''$$

and since the intensity of P before the screen was inserted was $I' = (p^2 \div q^2) I''$, we have for the transmission coefficient for the thickness t,

$$\alpha = \frac{I'_1}{I'} = \frac{p_1^2\ q^2}{p^2\ q_1^2}$$

45. THE PHOTOMETER.

To study the relation between intensity of illumination and distance from source, and to determine the candle power of an incandescent lamp and the transmission coefficient of tracing cloth.

Apparatus.—Photometer bench with attached scale and three sliding carriages, three incandescent lamps with leads and metal covers, photometer screen, two pieces of tracing cloth for transmission screens, and dark chamber to cover the whole apparatus.

DESCRIPTION.—The photometer bench consists of two heavy metal bars over two meters long, on supporting legs. Three metal carriages are free to slide along this bench, the middle one carrying the photometer screen, the other two bearing the sources of light to be compared. A two meter scale is fastened to the bench, and pointers on the carriages indicate their relative positions on this scale. The apparatus is screened from extraneous sources of light by a large dark chamber, painted dull black inside, which completely covers it except for a small opening in the cloth curtain front through which the photometer screen may be observed.

DIRECTIONS.—*Relation of intensity of illumination to distance.*—Place the two incandescent lamps of known candle power, P and Q, at opposite ends of the bench 200 cm. apart and move the Bunsen screen to such a position between them that the grease spot appears to have the same intensity of illumination as the rest of the screen. There will always be some difference in color between the spot and the rest of the screen, but both sides of the screen should appear alike when viewed in the mirrors. Use one eye so far as possible in judging this equality of intensities. The candle powers marked on the lamps represent the intensity in the direction of the indicator mark on the lamp. The lamps must therefore be placed so that this indicator mark faces the photometer screen. Note the distances of P and Q from the screen, and repeat with distances 150 cm. and 100 cm. between P and Q. Make three trials in each case and use the mean value for the position of the screen. With your data show that the intensity of illumination varies inversely as the square of the distance from the source.

Transmission coefficient.—Screen the lamp P by placing a sheet of tracing cloth between it and the photometer

srceen. Then with the lamps 150 cm. apart, move the photometer screen to the position of equal illumination and note distances p_1 and q_1. Repeat using two thicknesses of tracing cloth. Using the values p and q for the same distance between P and Q when P was not screened, calculate the transmission coefficient of the cloth and show how the intensity of illumination varies on transmission through more than one thickness.

Candle power of an incandescent lamp.—Replace the lamp P by the third lamp, of unknown candle power, with its indicator mark facing the screen, and move the photometer screen to the position of equal illumination. Note the distances p and q, and determine the candle power of this lamp in terms of that of Q.

Problems.—1.—A standard candle and a gas flame are placed $a(=10)$ feet apart, the gas flame being of $b(=6)$ candle power. Where would a photometer screen have to be placed between them in order to make the grease spot disappear?

2.—If the illumination at a point due to a certain source is $a=(6)$ candle feet, what will it be when a screen $b(=3.)$ mm. thick with a transmission coefficient of $c=(0.6)$ per mm. is placed between the source and the point?

3.—A light of intensity $a(=32)$ c. p. behind a sheet of glass which has a transmission coefficient $b(=0.5)$ per m.m. and a second light of $c(=18)$ c. p. at a distance $d(=5)$ meters, produce equal illumination on a Bunsen screen placed between them, and at a distance $e(=2)$ meters from the first light. What is the thickness of the glass?

4.—A lamp of $a(=16)$ c. p. is to be compared with a standard of $b(=8)$ c. p .when the distance between them is

$c(=2.)$ meters. If the Bunsen screen can be set for equality of illumination with an accuracy $\pm d(=5.)$ m. m., what is the % error in the determination of the c. p.?

REFLECTION.

THEORY AND DEFINITIONS.—D. 620, 642-652; G. 522-525, 530, 534-541; K. 815-828; W. 327-332, 337, 338.)

The simple laws of reflection may be stated as follows:

(1) *When a ray of light is reflected from a smooth surface, the incident ray and the reflected ray make equal angles with the normal to the surface and lie on opposite sides of it.*

(2) *The reflected ray lies in the plane of incidence, i. e., in the plane through the incident ray and the normal.*

By "ray" we mean here a line of propagation of a light wave.

When the rays of light coming from a point source are reflected, they may intersect each other at some point in front of the mirror before entering the eye, or they may diverge so that their prolongations will intersect at some point behind the mirror. In either case the eye preceives the image of the source at the point of intersection. In the first case the image is *real* and can be projected on a screen. In the second case it is called a *virtual* image and cannot be projected on a screen. Each point of a luminous object may be considered a point source and the images of these sources will together form an image of the object.

In order that we may express the relation between image and object distances from the mirror and its radius by a single equation, for all cases of plane and spherical mirrors, it is necessary to adopt certain conventions. We will call the center of the mirror, its *pole*, and the normal to the mirror

at the pole, or the line through the pole and the center of curvature, the *axis* of the mirror.

(1) *All distances are to be measured along the axis from the pole of the mirror.*

(2) *Distances measured from the pole in the direction in which the incident light travels are negative, those measured in the opposite direction are positive.*

According to these conventions, the radius of curvature of a plane or of a concave spherical mirror is positive and that of a convex spherical mirror negative; the distance of a real image is positive, that of a virtual image negative.

Calling the radius of curvature of the mirror "r," the distance of the source (or luminous object) from the pole "u" and the distance of the image from the pole "v," it may be shown that the relation between them for plane and spherical mirrors is

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the case of spherical mirrors this relation holds true only for *small apertures*, *i. e.*, when the diameter of the circular boundry of the mirror is small compared with its radius of curvature.

It is easily seen that a point source on the axis of the mirror will have its image formed on the axis or on the prolongation of the axis behind the mirror.

From equation (1) it is evident that in the case of a *plane* mirror where the radius of curvature r is infinitely great, we have

$$1/u+1/v=O, \text{ or } v=-u$$

That is, the image formed by a plane mirror is as far behind the mirror as the object is in front of it, and the image

is always *virtual.* Since in this case the normals to the different points of the mirror are all parallel, every point of the source may be considered as lying on an axis, and the image of each point will be on the prolongation of its particular axis. As a result the image will be *perverted;* that is, the right hand side of the object or source will appear as the left hand side of the image.

Since u and v are equally involved in equation (1) it is evident that in the case of real images, at least, the source and image distances are interchangeable; that is, a source at a distance u has its image formed by a mirror of radius r at a distance v, where u, v and r satisfy equation (1); if the source be placed at a distance v its image will be formed at a distance u. Two points so situated are called *conjugate foci.*

If the source be placed at the center of curvature, $u=r$ and we have

$$1/r+1/v=1/r, \text{ or } v=r$$

That is, the center of curvature is its own conjugate focus.

When *parallel* incident rays are reflected from a spherical mirror, the point of intersection of the reflected rays is called the *principal focus* of the mirror, and the distance of this point from the pole is the *focal length*, f, of the mirror. The focus (or source) of parallel rays is at an infinite distance, and putting $u=$infinity and $v=f$ in equation (1) we have

$$1/f=2/r, \text{ or } f=r/2$$

That is, the focal length of the spherical mirror is half its radius of curvature. For a concave mirror f is positive, for a convex mirror negative.

Replacing $2/r$ by its equivalent $1/f$, equation (1) becomes

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If we substitute different values of u in this equation we get the following relations *in the case of a concave mirror:*

for u = infinity, $v = f$, for $u > r$, $v < r$,
for $u = r$, $v = r$, for $u < r$, $v > r$,
for $u = f$, v = infinity, for $u < f$, v = negative.

Positive values of v and of f correspond to real images, negative values to virtual images, hence *the image formed by a concave mirror is real, except for values of u less than f, in which case it is virtual.* We see that as the source moves up from an infinite distance to the principal focus, the image moves off from the focus to an infinite distance, the two points meeting at the center of curvature.

In the case of the convex mirror, the value of v is always less than f and negative for positive values of u. If however the rays of light incident on the mirror converge toward a point $-u$ behind the mirror, v may become positive, and the image will be real.

Since the object and its image subtend equal angles at the mirror, it is evident that the relative sizes of both are in the ratio of their respective distances from the mirror or if O and I represent their corresponding dimensions,

$$\frac{O}{I} = \frac{u}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The apparent position of an image is sometimes obtained by the parallax method. *By parallax we mean the apparent displacement of an object (or image) relatively to another object, due to a real displacement of the observer.* If the two objects are coincident or are equally distant, their relative parallax vanishes.

When waves from a point source fall upon a spherical mirror obliquely, they do not form a point image, but there

will be formed two line images at different distances and at right angles to each other. The nearer the source is to the axis of the mirror, the more nearly these focal lines will coincide, and will be reduced to form a point image on the axis.

When the source of light is not a point, but an extended object, each point of this object will have its own point image formed, and all of these images together make up the image of the source. But those points which lie off of the axis of the mirror will have their images formed at different distances from the pole than that of the point on the axis. It will therefore be impossible to find a position of the screen at which all parts of the projected image are equally in focus.

46. MIRRORS.

To show that the angle of incidence is equal to the angle of reflection, and to find the position, size and shape of the image formed by a plane mirror; to show the relation between image and object distance and focal length of a concave spherical mirror, and to measure its focal length.

APPARATUS.—Optical bench, incandescent lamp with cover and slit, plane and concave spherical mirrors and holders, sliding blocks, right triangle, protractor, pins, screen with slit and cross wires, and sheet of paper.

DESCRIPTION.—The optical bench is a slotted channel iron bench about one and a half meter long, with a mm. scale attached to one side. (See Fig. 63.) The mirrors, lenses and screens to be experimented with are carried by metal blocks which are arranged to slide along the slot.

Their positions on the scale may be read by means of an indicator mark on the side of each block. One block carries a screen with one side enameled white so that images projected on it may be sharply focussed. It has also a rectangular opening in the center, across which cross wires are stretched. These may be used as the object to be focussed on the screen when a strong light is placed behind the opening. An incandescent lamp is covered with a copper screen, in one side of which is a rectangular slit. This generally serves as the object to be projected on the white screen, and it is this opening, not the incandescent filament of the lamp, which must be sharply focussed. To aid in focussing, a sharp spur is cut on one side of the slit. The mirrors and lenses are mounted so that they may be turned about both a vertical and a horizontal axis, in order to place them in any desired position, and the indicator mark on the carrying block is in all positions under the pole of the mirror or lens. On the front of these blocks are a pair of clamps for holding a small plane mirror for use in the parallax method of focussing.

Directions.—*Relation between angles of incidence and reflection.*—The plane mirror is fastened to one face of a supporting block. Draw a straight line across the sheet of paper and place the silvered (reflecting) side of the mirror along this line. Stick a black headed pin about 10 cm. in front of the mirror and a second ordinary pin to one side of the former and about 2 cm. in front of the mirror. Move the eye on a level with the mirror until a position is found where the images of the two pins coincide. Place a third pin *c* in line with the coincident images and the eye. Then draw the line through *a* and *b* to the reflecting surface and a perpendicular to this surface at the point of intersection *o*. Also join *c* and *o*. See Fig. 61. If the glass of the mirror

is thin the deviation of the rays *ao* and *co*, by refraction, may be neglected. Measure the angle of incidence and the angle of reflection and compare their values. Place the pins *a* and *b* in different positions and repeat, using large angles for the sake of accuracy.

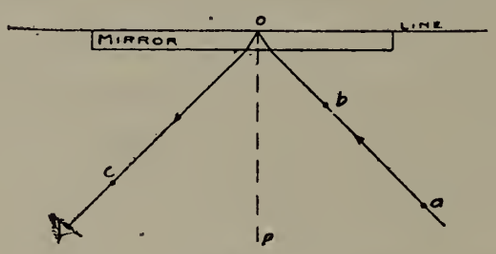


Fig. 61

Position, size and shape of the image in a plane mirror.— Draw a line on the other side of the paper, through its center and place the mirror as before. Lay the triangle in front of the mirror, trace its outline with a sharp pencil, and mark the angles *a*, *b* and *c*. Place a short pin at the vertex *a* and a large pin *a'* behind the mirror. *a'* is to be placed at the image of *a* by the method of parallax, hence *a'* must be moved to different positions until such a position is reached that upon turning the head from side to side or looking at the mirror from different positions, the image of *a* in the mirror coincides with the pin *a'* as seen above the mirror, for all angles of incidence. So long as *a'* is not at the image of *a* there will appear to be relative motion between these two as the head is turned from side to side. Locate the images *b'* and *c'* of *b* and *c* respectively in the same way. Join the points *a*, *b* and *c* to their corresponding images. Measure and compare the perpendicular distance of each vertex from the mirror, with that of its image. Join *a'*, *b'* and *c'* to form the image of the triangle. Measure and compare the lengths of its sides with that of the corresponding sides of the triangle.

Focal length of a concave spherical mirror.—Place the mirror on the optical bench facing the screen; carry them to a shaded window, and turn bench and mirror toward some *distant* object, such as a church tower, or a tree. Move the screen on the bench until a sharply outlined image of this object is obtained. Have the angles of incidence and reflection as small as possible. This image is formed at the principal focus. Measure the focal length between screen and pole of mirror.

Relation between distances of object and of real images from a spherical mirror.—Return the apparatus to the place assigned for performing the experiment. Arrange the lamp close to the bench so that the light passing through the slit may be reflected from the mirror to the screen, the slit being on the same level with the mirror and screen. To get sharper, though less brilliant images, the mirror may be covered by a diaphram made of paper or sheet metal, having the outline of the mirror and a circular hole at the center one to one and a half inches in diameter. The rectangular slit is the object whose image is to be formed and to aid in getting the image sharp a fine spur is cut so that it projects at one side of the slit. The distances u and v, of the slit and of the screen respectively from the center of the mirror may be measured on the scale. It should be remembered that for oblique incidence (which is unavoidable here) two images are formed at slightly different distances and a mean position must be found in each case.

Place the slit close to and in the same plane with the screen and move the mirror until a sharp image of the slit (not the lamp filament) appears on the screen. The distance between mirror and screen is now the radius of curvature of the latter. Measure this distance r and compare it with the focal length f.

This distance may also be determined by removing the disk with the slit from the lamp cover, placing the lamp directly behind the white screen, at one end of the bench and adjusting the position of the mirror until a sharp image of the cross wires is projected on the screen just beside the slit. The object and image are then both at the distance r from the mirror.

With the lamp cover and slit once more at the side of the bench, place the mirror at a distance between f and r, from the slit and move the screen until a sharp image of the slit is formed. Measure the distances u and v on the scale, and the length of the slit and of its image with the aid of dividers. Repeat with the mirror at a distance greater than r from the slit. Compare the sum of the reciprocals of u and v in the two cases with each other and with the reciprocal of the focal length f. Also compare the ratio of the object and image lengths in each case with the corresponding ratio of u and v.

Virtual image.—Remove the diaphram and fasten a short pin in the sliding block at a distance from the mirror less than the focal length, f. With a large pin behind the mirror locate the image of the short pin by the method of parallax, as described for a plane mirror.

Measure the distances u and v and, remembering the adopted convention concerning signs, compare the sum of the reciprocals of u and v with the reciprocal of f.

Owing to the spherical aberration, the image of the pin at the edges of the mirror curves from side to side as the eye is moved, and makes it difficult to determine these conjugate foci. When this curvature is objectionable the following method should be used: Place a small plane mirror M in front of the lower half of the spherical mirror, C. (See Fig. 62.) Place a black headed pin O at a distance less than f

in front of C, and a white headed pin P in front of M. By looking down at an angle at both mirrors, the image of O in C, (I), and of P in M, (Q), may be seen simultaneously without interference from the pins themselves. P must now be shifted in position until there is no parallax between the images I and Q, when the eye of the observer moves

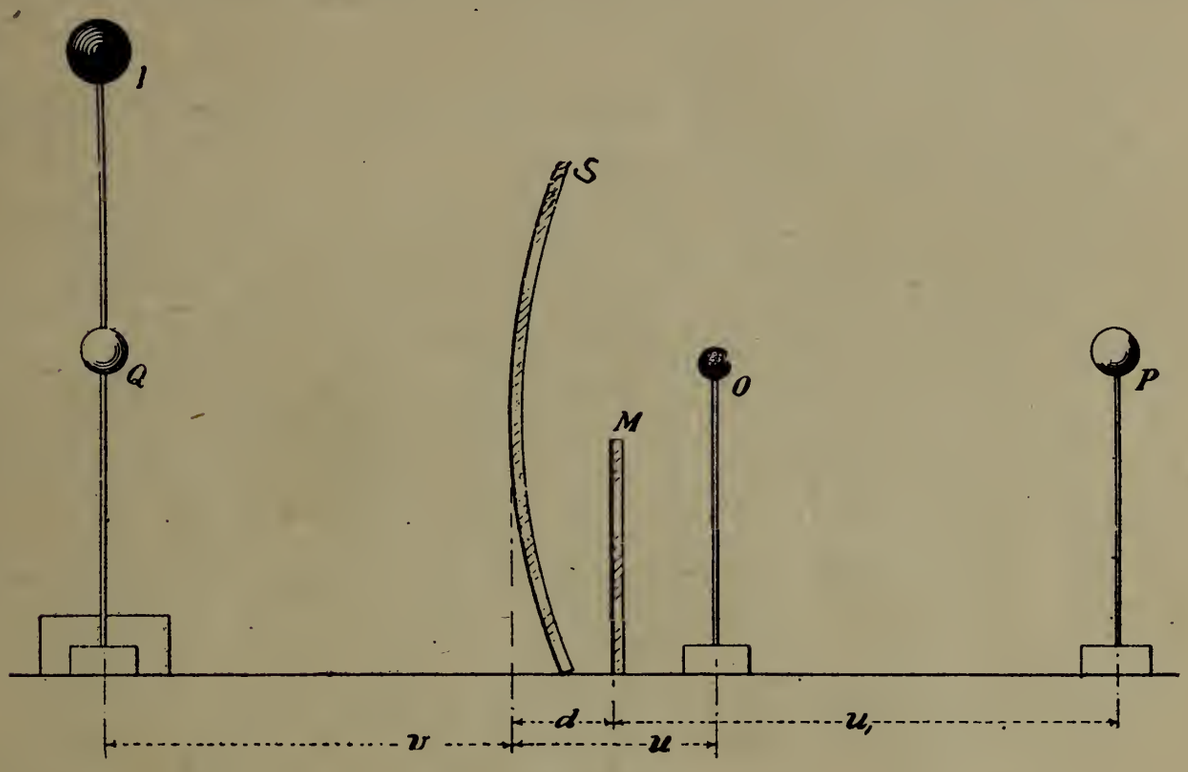


Fig. 62

from side to side. I and Q then occupy the same position, and Q is as far behind the reflecting surface of M as P is in front of it. Calling this distance "u_1" and the distance between reflecting surfaces of C and M "d," we see that the distance of I from C is

$$v = u_1 - d$$

Note on the scale the distances u, d and u_1, calculate v, and compare the reciprocals of f with the sum of the reciprocals of u and v.

Problems.—1.—Draw large clear diagrams showing the formation of a real, and of a virtual image of an arrow, by

means of a concave mirror, when the arrow is perpendicular to and symmetrical to its axis. Trace the path of two rays from each end of the arrow; one parallel to the axis of the mirror, or one through its center of curvature, and one through its principal focus. Mark center C, focus F, object O, image I, and the direction of the rays with arrow heads. Use full lines for actual rays, dotted lines for construction and prolongation of rays.

2.—In the same way draw diagram of image formation by a convex spherical mirror.

3.—*a*) A concave spherical mirror has a radius of curvature of $a(=30)$ cm. Find the position and size of the image formed when a candle flame with a height $b(=2)$ cm. is placed $c(=20)$ cm. in front of the mirror, on its axis. *b*) Find the position and size of the image when the candle is moved a distance $d(=8)$ cm. toward the mirror.

4.—A spherical mirror forms an image of an object having a height $a(=5)$ inches, and placed $b(=20)$ inches in front of the mirror. *a*) If the image is at a distance $c(=12)$ inches behind the mirror, find the focal length of the latter. *b*) Is the mirror concave or convex? *c*) What is the height of the image?

REFRACTION THROUGH LENSES.

THEORY AND DEFINITIONS.—(D. 620, 656, 666-667; G. 552, 562-572; K. 834-836, 849-858; W. 348, 349.) A lens is a portion of a refracting medium bounded by two curved surfaces, which are usually spherical. A line through the centers of curvature of the two surfaces is termed the principal axis of the lens. The points where this axis cuts the two surfaces are termed the poles of the surfaces. In the

following discussion the lenses are supposed to be thin; *i. e.*, the two poles are so close together that distances may be measured from the center of the lens. For definitions of aperture, real and virtual images, see previous theory of reflection.

Lenses are of two general kinds. Those which are thicker at the poles than at the periphery usually cause the incident light from a point source to converge after refraction to a point beyond the lens, when surrounded by a less refractive medium, and are called *convergent.* Those which are thinner at the poles than at the periphery cause the incident light to diverge after refraction from a point in front of the lens, when surrounded by a less refractive medium, and are called *divergent* lenses. When the incident light is parallel to the principal axis the lens will cause it to converge to, or diverge from a point called the *principal focus.* The distance from the lens to the principal focus is called the focal length of the lens. The focal length remains unaltered when the lens is reversed.

We shall adopt the same conventions for signs as in the case of reflection; namely:—

(1) *All distances are to be measured from the poles along the axis.*

(2) *Distances measured from the poles in the direction in which the incident light travels are negative, those measured in the opposite direction are positive.*

According to this convention the focal length of a convergent lens is negative, that of a divergent lens positive.

Let u be the distance of an object from the lens center, v the distance of its image, and f the focal length. Then for lenses having a *small aperture*, the following relation holds for both kinds of lenses:—

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

It should be remembered that besides the signs of the fractions in this equation each symbol (u, v, f,) has its own sign depending upon the conventions given above.

This equation gives us the following relations:—

Convergent lenses:—

For u = infinity,	$v = -f$.
For $\infty > u > 2f$,	$-f > v > -2f$.
For $2f > u > f$,	$-2f > v > -\infty$.
For $f > u > o$,	$\infty > v > o$.

In the last case the image is virtual. The other images are all real.

Divergent lenses.—

For $u = \infty$,	$v = f$.
For $\infty > u > o$,	$+f > v > o$.

The image formed by the divergent lens is always virtual and always nearer to the lens than the object.

Since the object and image subtend equal angles at the center of the lens, their sizes will be in the same ratio as their respective distances from the lens. Hence, if O and I represent their corresponding dimensions

$$O/I = u/v.$$

If a lens forms an image of an object on a screen when it is half way between them, it may be shown that the distance between them is four times its focal length, f. If the fixed distance between object and screen is greater than $4f$, there will be two positions of the lens between them, at which

it forms a distinct image of the object on the screen. This fact suggests a method of determining the focal length of the lens. For if s be the distance between the object and the screen (greater than $4f$), and l be the distance between the two positions of the lens, at which it projects a sharp image on the screen, it may be shown that the focal length is

$$f = \frac{s^2 - l^2}{4s}$$

47. LENSES.

To study the relation between focal length, image and object distance, and image and object size in the case of convex and concave lenses, and to measure their focal lengths.

Apparatus.—Optical bench, double convex and double concave lens and stand, incandescent lamp and leads, cover and slit, sliding screen, two sliding blocks meter stick and pins. (Apparatus described in Exp. 46.)

Directions.—*Convex lens.*—Take the optical bench to a shaded window. Place the convex lens in its holder at one end of the bench with its axis pointed toward some *distant* object, such as a chimney or church steeple. Move the screen along the bench until a sharp image of the object is projected on it. The rays from the object are practically parallel and the distance from lens to screen is the focal length of the former. Measure this length on the attached scale.

Place the lamp with its slit vertically over zero of the scale and place the lens at a distance between f and $2f$ from the slit. Fig. 63. Then move the screen until there is a sharp image of the slit (not of the lamp filament) upon it.

Adjust for sharp definition by observing the spur on the side of the slit. Measure the distances u and v and also the lengths O and I of the object and image. Repeat with the

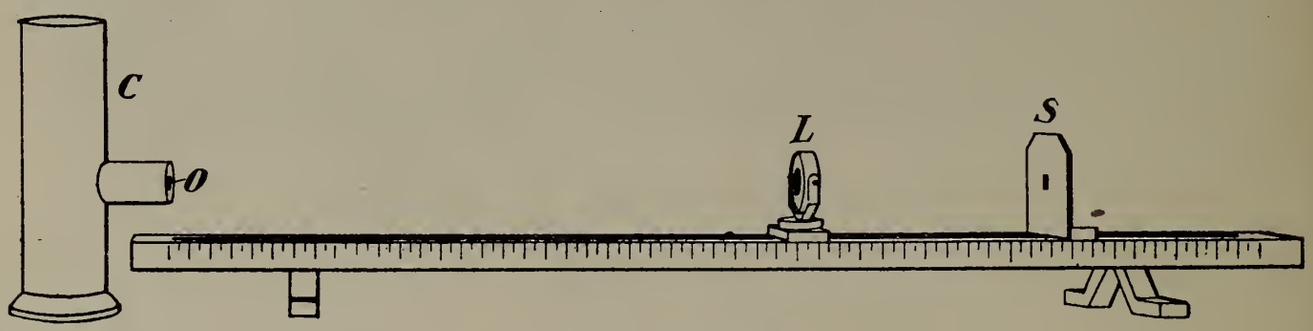


Fig. 63

lens at a distance equal to $2f$ and then at a greater distance from the slit. Compare $-1/u+1/v$ with $1/f$ in both cases, remembering the adopted conventions regarding signs. Also compare O/I with u/v.

Virtual image.—Place a block with a small pin behind the lens at less than the focal distance f. Locate the virtual image of this pin by the method of parallax; *i. e.*, place a second long pin on a sliding block behind the first pin and move the second on the bench until it is at such a position that the image of the first pin seen through the lens, and the top of the second pin seen above the lens coincide for all positions of the eye in front of the lens. (The second pin is then at the virtual image of the first pin.) See Fig. 64. Measure the distances u and v and compare $-1/u+1/v$ with $1/f$.

If the curvature of the image at the edge of the lens due to spherical aberration is appreciable, it is better to place a small plane mirror in front of the lens, and proceed as described in Experiment 46, to find the virtual image. The only change in this method is that the pin O is on the same side of the lens with the two images I and Q, and its image is seen through the lens.

The distances u, d and u_1, are to be measured and v calculated, in order to substitute u and v in the lens equation.

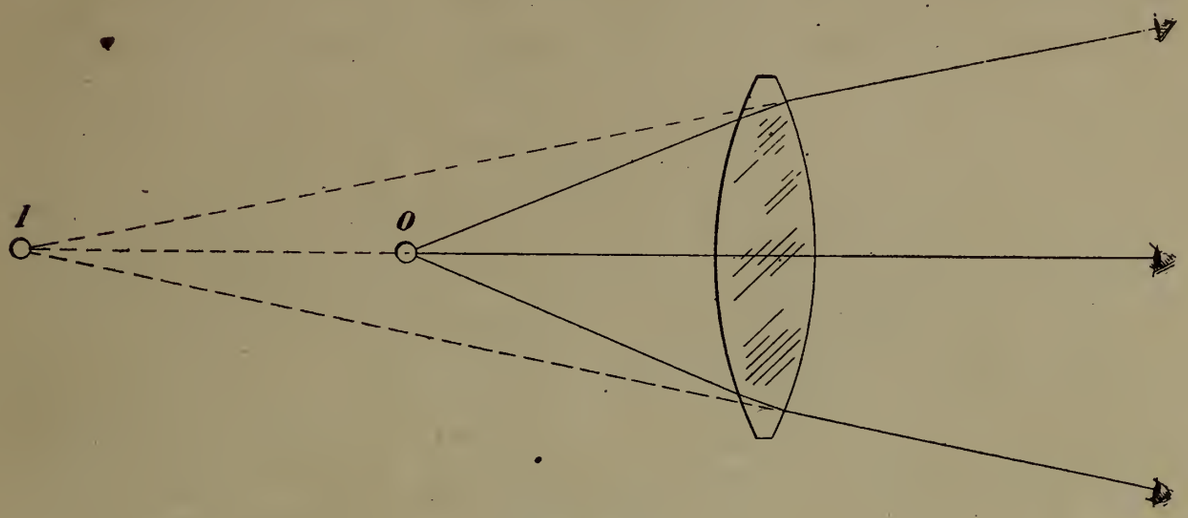


Fig. 64

Concave lens.—The images are all virtual and are located by the parallax method. Take the optical bench to a shaded window and locate the image of a distant vertical line such as the corner of a building, by sliding a long pin along the bench, behind the lens, until it coincides with the image of the object for all angles of observation. Measure f from the lens to the pin. As in the previous case of a virtual image, so here also, it is preferable to use the parallax method described in Experiment 46.

Place a second pin at a distance $2f$ from the lens and locate its image. Measure u and v. Repeat with the pin at a distance f from the lens. Compare $-1/u+1/v$ with $1/f$.

Problems.—1.—Draw large, clear diagrams showing the formation of a real and of a virtual image of an arrow by means of a double convex lens, the arrow being perpendicular to and symmetrical to the principal axis. Trace the path of two rays from each end of the arrow; one parallel to the principal axis, and one through the principal focus, or one through the center of the lens. Mark object O, image I,

principal focus F, and direction of rays by means of arrow heads. Use full lines for actual rays, dotted lines for construction and prolongation of rays.

2.—Lay off on coordinate paper a pair of rectangular axes intersecting in the upper left hand corner. On these axes lay off the corresponding values of u and v obtained above for the real images of a convex lens as abscissae and ordinates respectively. Connect corresponding values of u and v by straight lines. Compare the ordinates of the common point of intersection of these lines, with each other and with the focal length of the lens as determined experimentally above. Using the lens equation, show by the proportion between the sides of the similar triangles formed by one of the oblique lines, that the ordinate of this point should equal the focal length of the lens.

3.—An object which is $a(=20)$ inches high is looked at through a double concave lens of $b(=15)$ inches focal length when it is $c(=30)$ inches from the lens. Find the size and position of the image formed.

4.—A double convex lens of $a(=8.6)$ cm. focal length casts a sharp image of a lamp on the screen, when placed between them at a certain point. When it is moved a distance $b(=15)$ cm. nearer to the screen it again forms a sharp image. Find *a*) the distance between lamp and screen, and *b*) the distance from lamp to lens in its first position.

48. INDEX OF REFRACTION OF A LENS.

To determine the index of refraction of a lens from its focal length and the radii of curvature of its faces. (D. 668; G. 539, 545; K. 849-851; W. 351, 352.)

APPARATUS.—Optical bench, double convex lens, spherometer, piece of plate glass, millimeter scale.

Description and Theory.—When parallel rays of light fall on a double convex lens they converge to a point called the principal focus of the lens. The reciprocal of the distance f, from the center of the lens to this point is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1)$$

(see references above for development) where r_1 is the radius of curvature of the face upon which the light impinges, r_2 the radius of curvature of the other face and n the index of refraction of the lens.

The focal length of the lens may be obtained by allowing

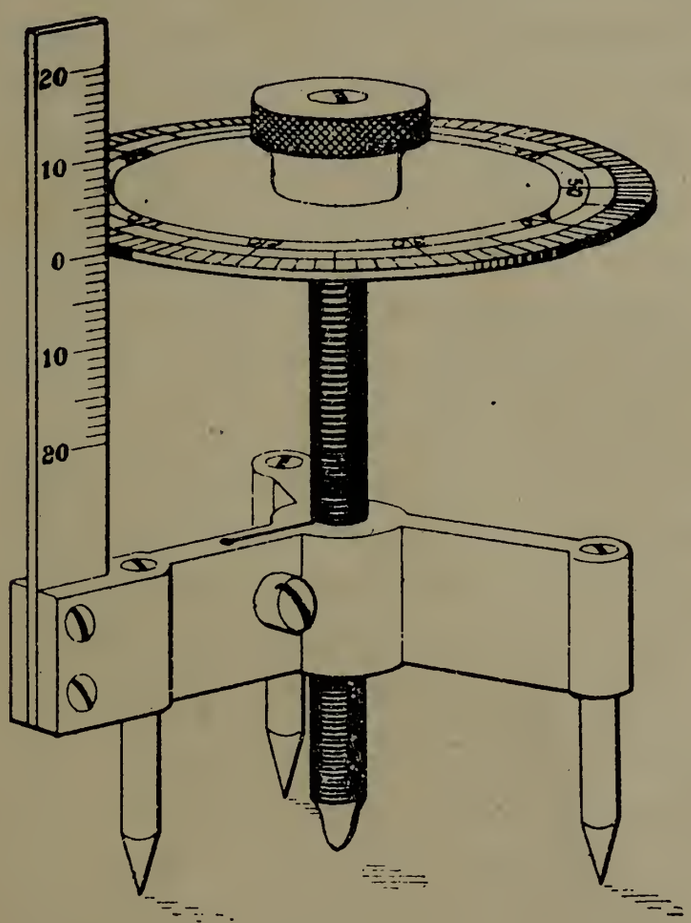


Fig. 65

rays of light from a distant object (tree, building, etc.) to fall on the lens and be brought to a focus on a screen placed behind the lens. When a clear image is obtained, the dis-

tance from the screen to the center of the lens will give the focal length *f*. The radii of curvature of the two faces of the lens may be obtained by use of the spherometer.

The spherometer consists of a screw of one millimeter pitch which turns in a nut supported by an equilateral tripod. (See Fig. 65). The axis of the screw passes through the center of the circle in which the three feet of the tripod are situated. The whole number of turns made by the screw may be observed from the vertical millimeter scale erected over one of the legs of the tripod. At the head of the screw is a circular disk which is divided into 100 equal parts. Since the pitch of the screw is one millimeter, one complete turn represents a longitudinal motion of the screw of one millimeter and turning the screw through one of the small divisions on the disk represents a longitudinal motion of the screw of one-hundredth of a millimeter. If the plane of the disk is between 6 and 7 on the vertical scale and the forty-fifth division of the disk is in line with the face of the vertical scale, the reading is 6.45 *mm*. Tenths of a division on the disk may be estimated.

To find the radius of curvature of a spherical surface, it is first necessary to determine the position of the disk when the three legs and the point of the screw are in the same plane. This is done by placing the spherometer upon the glass plate and turning the screw down till its point rests upon the glass. Then raise the screw and, placing the spherometer on the convex lens, turn the screw down again until it comes in contact with the glass at the point *e*, Fig. 66. The difference between this reading and the "zero" reading is the amount the point of the screw has been removed from the plane of the ends of the three legs. Let this distance be represented by *a*, the distance between the legs and the point of the screw (when the points of the screw

and the legs are in the same plane) by b, and the radius of curvature by R. It may be seen from Fig. 66 that

$$R^2=(R-a)^2+b^2 \qquad (2)$$

Solving this for R gives

$$R=\frac{a^2+b^2}{2a}=\frac{a}{2}+\frac{b^2}{2a} \qquad (3)$$

Directions.—(1) To find the radii of curvature of the faces of the lens, first place the spherometer on a sheet of plate glass and turn the screw down till it touches the plate. To test for this adjustment, apply a very slight pressure immediately above each of the three legs and test to see

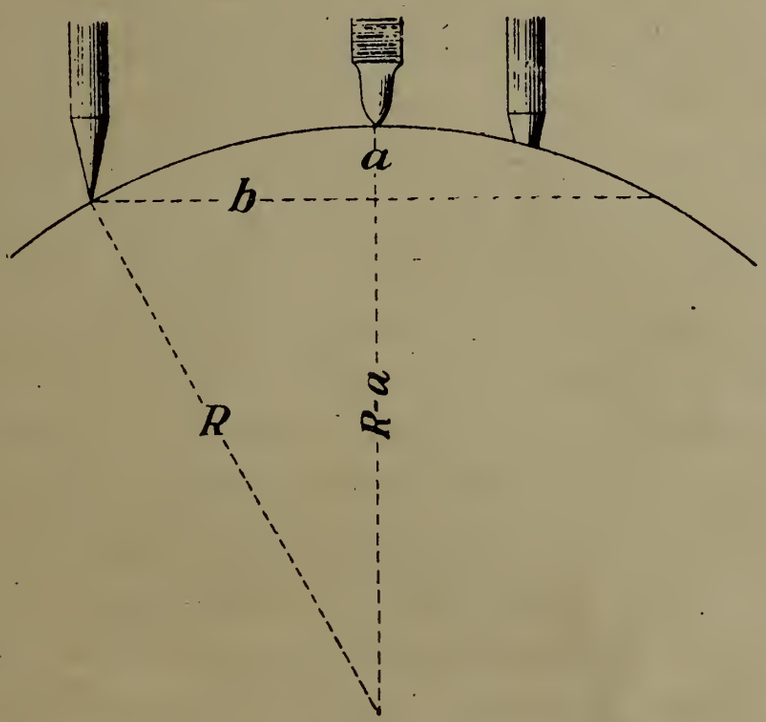


Fig. 66

whether the spherometer rocks on the point of the screw. Adjust the screw until the position is found where this motion just disappears. Read the position of the top of the disk in millimeters by means of the vertical scale and the fraction of a millimeter by means of the divisions on the circular

disk. The tenths of the small divisions can be estimated by the eye. Move the spherometer to other positions upon the plate and make 10 readings as above, for the "zero position" of the screw. Take the mean of the 10 readings for the true value of the "zero position." Raise the screw, place the spherometer on the convex lens and then turn the screw down till the point makes contact in the same way as it did on the glass plate. Note the reading as indicated by the vertical scale and the divided disk. Move the spherometer to different positions, make 10 different settings of the screw and record the reading for each setting. Take the mean of the 10 readings and find the difference between it and the mean "zero position." This difference is the distance a in the formula above and is in millimeters. Record the difference in centimeters. To find the distance b between the legs and the point of the screw, place the spherometer upon a piece of paper on the plate glass and make the impression of the points of the legs and screw upon the paper. With a millimeter scale measure the distance between the central impression made by the screw and the inside part of the impressions made by the legs. The inside of the impressions made by the legs is chosen because, the surface being convex, the inside of the end of each leg is the part that rests upon the surface. Make two such sets of impressions and find the mean value of the 6 determinations of the distance from the central impression to the outer ones made by the legs. Call this mean value b and record it in centimeters. From the values of a and b thus obtained, calculate the value of the radius of curvature from equation (3).

In like manner find the radius of curvature of the other face of the lens.

(2) To find the focal length f, place the lens in the holder on the optical bench and adjust the screen so that rays from a

distant object come to a focus on it. Measure the distance from the screen to the center of the lens and record it in centimeters. Make five settings of the screen and take the average as the value of f.

Substitute the values of f, r_1 and r_2 in equation (1) and solve for n, the index of refraction of the lens. Care must be taken to assign to r_1 and r_2 their proper algebraic sign.

Questions.—Draw figure and derive formula for radius of curvature R, of a *concave* surface. What distance should be taken as b? Give reason for your answer. If an error of .5 *mm* were made in measuring b, what per cent error would it make in the value of R in your results?

ALTERNATIVE METHOD

When a spherometer is not available for measuring the radii of curvature of the lens, the following method may be used:

Theory and Method.—If light from a source O falls on a double convex lens placed so that the light is reflected from the back surface of the lens and forms an image of the source at the same point O, it is evident that within the lens the rays are perpendicular to the back surface, otherwise they would not retrace their incident path. This image is in exactly the same position as it would have been if it were formed by light waves falling perpendicularly upon the back of the lens and converging toward the point of intersection of the projection of the rays within the lens, namely P. This point is also the center of curvature of the reflecting surface. (See Fig. 67.) The distance PC will be the apparent object distance u of such waves, and OC the corresponding image

distance v. If f be the focal length of the lens we have

$$1/f = -1/u + 1/v,$$

and as v and f are both measurable quantities we may calculate the distance PC from this formula. If the lens is very thin this distance is practically the radius of curvature r_2 of its back surface, but for an ordinary lens we must add to this one half the thickness of the lens at its center.

The lens is then reversed and the radius r_1 of its other face is determined in the same way. Knowing the focal length and the radii of curvature of the lens faces, the index of refraction may be calculated by the formula already given.

DIRECTIONS.—With the screen and cross wires at one end of the bench, place the lamp behind the screen and move

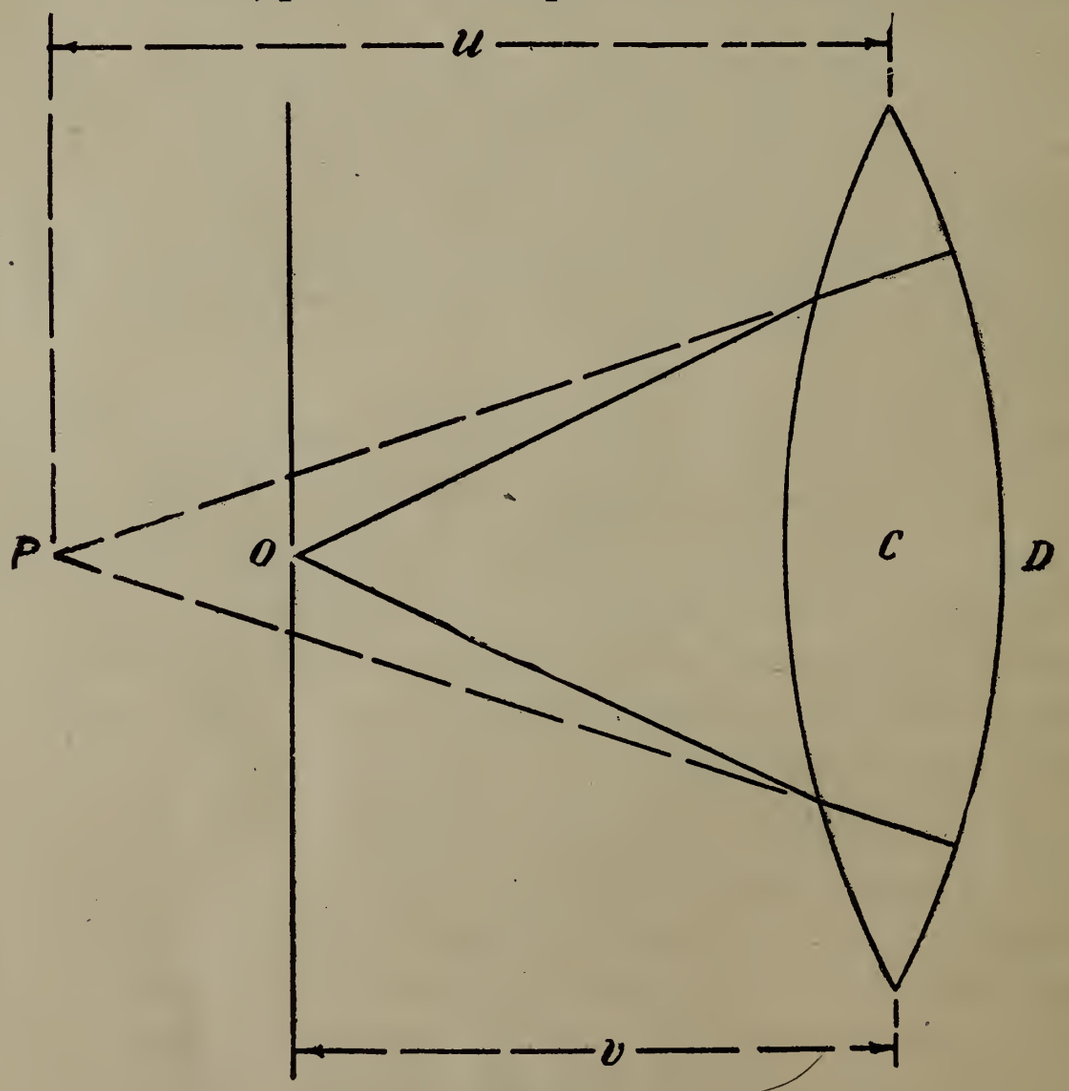


Fig. 67

the lens along the bench in front of it until a sharp reflected image of the wires is projected on the screen. By turning the lens slightly the image may be seen just beside the opening in the screen. Note the distance v between the cross wires and center of lens. Then reverse the lens and repeat the measurement. With vernier or micrometer calipers measure the thickness of the lens at its center, taking the mean of five or six measurements. From these data calculate the radii r_1 and r_2. Next determine the focal length of the lens by one of the methods described in Experiment 47. Finally calculate the index of refraction by formula (1).

Question.—What % error would result from neglecting the thickness of the lens in calculating the radii of curvature?

49. THE CRITICAL ANGLE FOR GLASS IN AIR

To determine the index of refraction and the critical angle for glass in air. (D. 656-678; G. 547, 550; K. 835, 840, 841; W. 343, 344.)

Apparatus.—Glass cube, straight edge, long pin, and sheet of paper.

Theory and Method.—When a light ray passes from one transparent medium into a denser one at any angle of incidence except 0° or 90° it will be bent toward the normal to their interface. But if the refraction takes place from the dense into the rarer medium, the ray will be bent away from the normal. In other words the angle of refraction increases more rapidly than the angle of incidence as we increase the latter from 0°. It is evident that the angle of refraction can not exceed 90°, hence for some angle of incidence less

than 90° refraction will cease and the incident ray will be reflected back into the denser medium. This angle of incidence is called the *critical angle*, and the reflection beginning at this angle is called *total reflection*. The critical angle evidently depends upon the values of the index of refraction for the media, and as these vary with the wave length of the incident light, the critical angle also varies with the color of the light. We may however determine an average value for white light.

When the angle of incidence has reached its critical value α, the angle of refraction is 90°, and by Snell's law we have for the index of refraction (n being always taken for incidence in the rarer medium,)

$$1/n = \sin \alpha / \sin 90°, \text{ or } \sin \alpha = 1/n.$$

Hence if we measure the index of refraction of a substance we may calculate its critical angle by means of this relation or vice versa.

DIRECTIONS.—Place the glass cube *ABCD* near the right hand upper edge of a sheet of paper on the table, and draw its outline with a sharp pencil. (See Fig. 68.) Stick a vertical pin *P* in the table close to the glass and near its corner *B*. The glass must not be moved from this position during the experiment. Lay the straight edge on the paper in front of the glass and place it so that when sighting along its edge the pin can be seen through the glass. Then draw a straight line *FE* along its edge. Shift the ruler to a different position at which the pin may be seen along its edge, and in a similar manner draw five such lines as *FE*. Remove the glass and prolong these lines to meet *DC*. Then join the point *P* to the intersection of each line with *DC*, and prolong these incident rays to some convenient length, say

10 cm. The index of refraction may then be determined for each pair of incident and refracted rays as follows:

Lay off an equal length (10 cm.) along the refracted ray EF, draw the normal to the surface DC at the intersection E, and drop perpendiculars from the extremities F and G

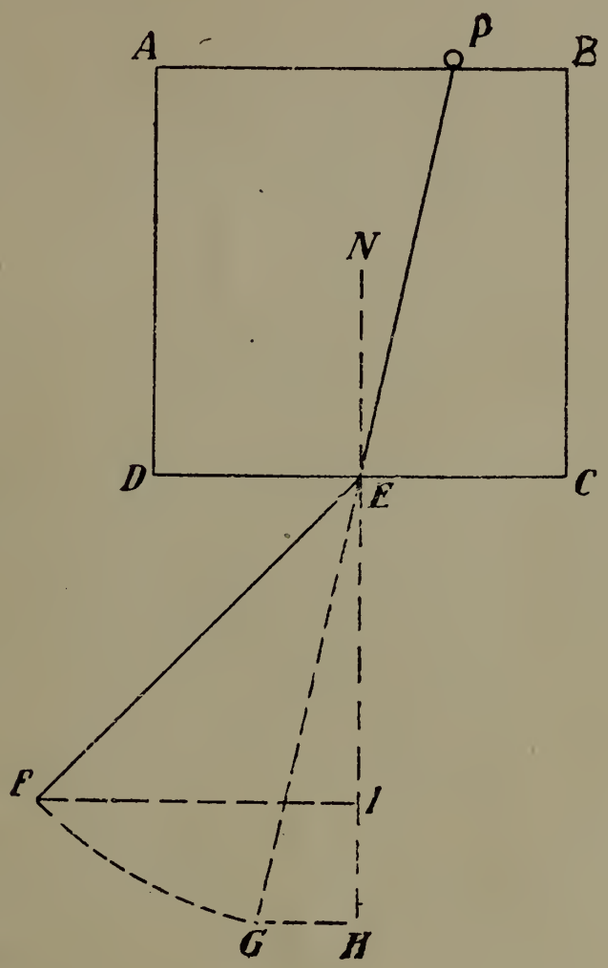


Fig. 68

of these lengths upon the normal NE. The sine of the angle of incidence is then GH/GE, (for the angle GEH equals the angle PEN.) and the sine of the angle of refraction is FI/FE or FI/GE. Hence the index of refraction is by Snell's law

$$n = \sin r / \sin i = FI/GH.$$

From the mean value of n calculate the value of α by the equation given.

QUESTIONS.—*a.*) Explain why a right angled prism may be used to change the direction of a beam of light at right angles to its original direction, when it impinges perpendicularly upon one of the smaller faces. *b.*) Would the prism act in the same way if it were immersed in carbon bisulphide ($n = 1.63$)?

ARRANGEMENT OF LENSES IN MICROSCOPES AND TELESCOPES.

THEORY AND DEFINITIONS.—(D. 700-705; G. 597-608; K. 882-891; W. 353-355.) Any convex (convergent) lens may be used as a simple microscope or magnifying lens. With the unaided eye an object is most distinctly seen at a distance of about 25 cm. When the object is observed through the lens placed at less than its focal length from the object, the angle which it subtends at the eye is increased and a magnified virtual image is seen. The magnifying power is defined as the ratio of the apparent size of the image to the size of the object as seen at the distance of most distinct vision, (25 cm.). Though the object is always a little nearer the lens than the focus, if the eye is placed very near the lens, the approximate value of the magnifying power will be 25 cm. divided by the focal length f.

In the *compound microscope* at least two convex lenses are used. One of these, the objective, is placed so that the object lies just beyond its principal focus and it forms an inverted and magnified real image. The second lens, the eye piece, is placed so that this real image falls just inside of its focus, and forms a more magnified virtual image of the first inverted image. The magnification by the objective is the ratio of image to object distance v/u, and that by the eye piece is $25/f$, whence the total magnification is $25v/fu$.

The *astronomical telescope* consists of at least two convex lenses. The objective forms an inverted reduced real image of a distant object. This image when viewed through the eye piece appears as a magnified virtual image which is still inverted. To form an erect image one or more additional lenses would be needed.

In the *Galilean telescope* the image formed by the convex objective is viewed through a concave (divergent) eyepiece, which is so placed in front of the image that the distance between the two lenses is the difference between their focal lengths.

The magnification in either kind of telescope is the ratio F/f where F is the focal length of the objective, f that of the eyepiece.

In practical microscopes and telescopes both eyepiece and objective are built up of several lenses in order to give the proper focal length and to overcome spherical and chromatic aberration, etc., the lenses being mounted in telescoping tubes.

50. MICROSCOPE AND TELESCOPE.

a. To find the magnifying power of a simple microscope.

b. To arrange a pair of lenses for use as a compound microscope.

c. To arrange a pair of lenses for use as an astronomical telescope.

d. To arrange three lenses for use as a terrestrial telescope.

e. To arrange a pair of lenses for use as a Galilean telescope.

APPARATUS.—Optical bench sliding screen with fine printing on it, screened light, one large double convex lens, two small convex lenses, one small double concave lens, holder for each lens, sliding cross-hairs, millimeter scale, pins.

DIRECTIONS.—*a.*—Place the large convex lens at one end of the bench. Adjust the mounted cross hairs behind the lens until they are sharply focussed. Place the screen at a distance of 25 cm. behind the lens. Looking at the parallel sides of the opening in the cardboard on which cross hairs are mounted, *through* the lens and at the screen *above* the lens place pins in the screen at the position of these parallel sides. Measure the distance between lens and cross hairs and between lens and screen, also width of opening in cross hair mount and distance between pins. The width of the image may be measured more directly by placing a *mm.* scale in position of the screen and observing the cross hairs and slit through the lens with one eye, the scale through the air with the other eye. The position of the scale should be adjusted until the image is seen apparently projected on the scale, so that the position of the sides of the slit on the scale can be noted.

The ratio of the image width to the object width is the magnification at the distance of distinct vision. Compare this with the ratio between the distance from lens to screen and from lens to cross hairs. Repeat the above measurements.

b. Compound Microscope.—Determine the focal lengths of the different lenses. (See Exp. 47). Place a small convex lens *a*, at a little more than its focal length from the slit in the light screen, using it as the objective. Adjust the sliding screen until an image of the slit is sharply outlined on it. Measure the length of both the slit and its image, and the

object and image distances from the lens (u and v). Note whether image is erect or inverted. Place the other small convex lens b, (the eyepiece), back of the screen at a distance less than its focal length. Remove the screen, adjust the lens b until the image seen through it is sharply outlined and note whether it is erect or inverted. Clamp the meter stick in a vertical position 25 cm. back of lens b and just beside the bench so that it does not interfere with the light between the lenses. With one eye look through the lens at the slit, then open the other and look through air at the meter stick, adjusting the position of the latter until the image of the slit is seen apparently projected on the scale. The ratio of the image length at 25 cm. to the slit length is the magnification. Compare this with the magnification calculated from u, v and f (focal length of eye piece). Compare the length of slit with that of its two images. Observe that the image formed by b cannot be projected upon a screen. Repeat the above measurements using the large convex lens c for the objective.

c. Astronomical Telescope.—Place the large convex lens c about 60 to 100 cm. from the slit and locate the position of the image by means of the screen. Measure the lengths of slit and image and note whether the image is erect or not. Place the convex lens a back of the image at less than its focal length, remove the screen and observe the image through the lens a. Note whether the image formed by a is erect of not. Adjust a until the image appears sharp, and measure the length from c to a. Place the cross hairs back of a and adjust them until their image appears sharp. Compare the position of the cross hairs with the position of the image formed by c.

To determine the magnification place a vertical meter stick from 5 to 10 meters in front of lens c and observe it

and its image simultaneously as explained for the microscope. Adjust the meter stick until its image is projected upon it, and note the position on the scale, of two marks on the image. The ratio of the distance between the two positions to that between the two marks will be the magnification. Compare this with the magnification calculated from the focal lenghts of objective and eyepiece.

d. Terrestrial telescope.—Leaving *c* in position move *a* about twice its focal length from the image formed by *c*. Observe that a real image is now formed by *a*. Note its position and length and whether it is erect or not. Place the third convex lens *b* back of the second image at a distance less than its focal length. Observe the image through *b* adjusting the latter until it forms a sharp image and note whether this image is erect or not. Measure the length from *c* to *b*. Determine the magnification as before, and compare the lengths of the slit and the three images.

e. Galilean telescope.—Leave *c* in position, but remove *a* and *b*. Place the concave lens *d* in front of the image formed by *c*. Observe that no real image is formed by *d*. Adjust *d* until the image seen through it is sharp. Measure the length from *c* to *d*, and compare it with the lengths of the other two telescopes. Determine the magnifying power as with the astronomical telescope and compare it with the value calculated from the focal lengths of the lenses.

PROBLEM.—Draw diagrams showing the formation of the images in each of the above five cases.

51. THE SPECTROMETER.
INDEX OF REFRACTION OF A PRISM.

To determine the index of refraction of a glass prism from the prism angle and angle of minimum deviation, by means of a spectrometer. (D. 659, 660, 709; K. 557-561; G. 835, 847, 848; W. 345-347, 357, 367, 369.)

Apparatus.—Spectrometer and prism, incandescent lamp, hood, and black cloth.

Theory; Definitions and Description.—The index of refraction at the interface between two media is the ratio of the velocities of light in the two media, or the ratio of the sine of the angle of incidence to the sine of the angle of refraction. This ratio is according to Snell's law a constant for all ordinary substances for a given wave length of light. As usually tabulated this index is given for different substances in air, the angle of incidence being taken in air. If a prism of the substance is available, its index of refraction may be determined with the aid of a spectrometer.

Let A be the angle of the prism between the two faces which intersect in the refracting edge and D the angle of minimum deviation, *i. e.*, the least angle which is formed by any ray between its direction before entering the prism and its direction after leaving the prism. It may be shown that the index of refraction is

$$n = \sin\frac{A+D}{2} \Big/ \sin\frac{A}{2}$$

A and D may be measured on the spectrometer. The spectrometer consists of an achromatic telescope, a graduated circle, and a collimator supported on a heavy tripod base. The collimator is a device for producing a beam of parallel

rays. It consists of an adjustable slit and a convex lens, at opposite ends of two telescoping tubes. In some forms of spectrometer the three essential parts of the instrument may be rotated about a common center and may be clamped in any position relative to each other. The collimator and telescope may also be rotated about vertical and horizontal axes perpendicular to their optical axes and are provided with slow motion tangential screws for adjusting their position accurately. The circular scale is provided with two verniers and reading glasses 180° apart, for accurate reading. A small adjustable table at the center of the graduated circle serves to hold the prism, grating or other device to be used.

DIRECTIONS.—*To Adjust the Spectrometer.*—Focus the telescope for parallel light by placing the cross hairs in an oblique position, adjusting the eyepiece until they appear sharp, and then sighting the telescope on a *distant* object, adjusting the distance between eyepiece and objective (without altering the relative position of eyepiece and cross hairs) until there is no parallax between the intersection of the cross hairs and the image of the distant object.

Next level the instrument, remove the prism table, and place a vertical lead pencil in its stead. Adjust the telescope and collimator until their axes are in the same horizontal plane and in the vertical plane through the center of pencil. When this is properly done the intersection of the cross hairs will lie on the center of the slit. If it is necessary to rotate either instrument in further adjustments, be careful to grasp them by their arms not by their tubes, so that they are not accidentally turned about their own vertical axis. Next place a light (incandescent lamp) behind the slit and sight through telescope and collimator at it. Let the slit be quite narrow but distinctly visible and adjust the distance

between it and the collimator lens until there is no parallax between slit and cross hairs of the telescope. The prism table is then replaced and the prism set in position with its refracting edge parallel to the axis of the spectrometer and its center in line with the axis of telescope. The planes of the vertical faces of prism must be as nearly perpendicular as possible to the plane passing through the optical axes of the lens systems of telescope and collimator. Ask the instructor for assistance in doing this.

To measure the angle of the prism.—To avoid possible errors in interpreting readings, clamp the zero of the circular scale under the collimator *C*. (See Fig. 69.) Turn the prism *P* until its refracting edge is pointed directly at the center of the collimator lens and clamp it in position with this edge over the center of the spectrometer. Illuminate the slit by means of the incandescent lamp and adjust its width so that it is very narrow, yet distinct. Turn the telescope *T* about the center of the circular scale, until the image of the slit reflected from the right face of the prism is seen. Clamp the telescope in position and by means of the tangential screw set the cross hairs on the right edge of this image. Read the position on the verniers. Repeat this

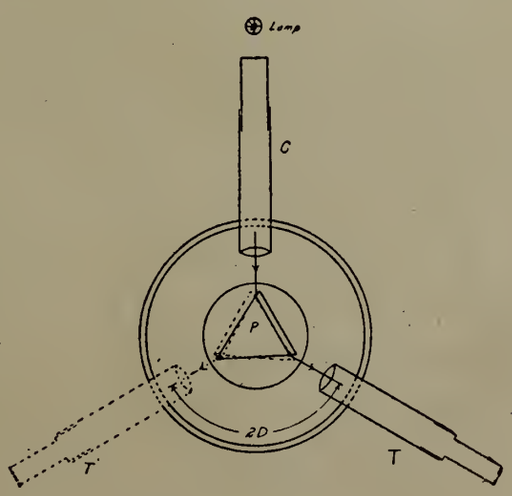


Fig. 69

performance with the image reflected from the left face of the prism, setting the cross hairs on the right edge as before. It may be shown that the angle A is half the angle between the two positions of the telescope. Alter the position of the prism slightly and repeat this determination as a check.

To measure the angle of minimum deviation.—Place the prism with its center over the center of the spectrometer, (the base being nearer to the center than the refracting edge,) and clamp it in position. If the beam of light is incident at the angle of minimum deviation its path through the prism will be symmetrical with respect to the two polished faces of the prism. When the slit is illuminated by the incandescent lamp the prism forms a spectrum. To get a sharply defined image of the slit it is necessary to use a source of monochromatic light (of a single wave length) such as the sodium flame. Remove the lamp and illuminate the slit with a Bunsen burner, on the tip of which has been placed an asbestos tube impregnated with sodium chloride. An intense yellow flame is produced. Turn the prism table until the refracting edge points at right angles to the collimator tube and locate the refracted image of the slit by eye. Turn the telescope until the image may be observed through it. Then gradually turn the refracting edge away from the collimator. The image will turn toward the collimator. Follow the image with the telescope until it just begins to move in the opposite direction. Clamp the telescope in this position, and turn the prism table slightly first in one direction, then in the other, until the image is just at its turning point. Then adjust the cross hairs on the image by means of the tangential screw, and read the verniers. Next remove the prism, place the telescope in line with the collimator, clamp it, adjust the cross hairs to coincide with the image of the slit by means of the tangential screw and

read the verniers. The difference between the two positions of the telescope is the angle of minimum deviation D. Alter the position of the circular scale slightly and repeat the determination.

Calculate the value of n from the mean values of A and D.

Problems.—1.—Draw a diagram and show the relation between A and the telescope positions for determining it.

2.—Draw a diagram and derive the expression for the index of refraction as given under theory.

52. THE DIFFRACTION GRATING.

To determine the wave length of the D lines of the sodium spectrum with the aid of a diffraction grating. (D. 696; G. 660-663; K. 935-940; W. 267, 374.)

Apparatus.—Spectrometer, diffraction grating, sodium light, incandescent light and black cloth.

Theory and Method.—The diffraction grating is a plate of glass or of polished speculum metal on which a series of fine parallel lines (usually several thousand in the space of a centimeter) are ruled. When light passes through the glass or is reflected from the metal these lines form spectra by diffraction. A replica of these gratings in gelatine or other transparent substance, placed between two parallel sheets of plate glass, acts in the same way. The method of producing the spectra may be explained with the aid of figure 70, in which the cross hatched spaces represent the opaque ruled lines and the spaces between them are transparent glass or gelatine. If a plane wave of monochromatic light from a narrow slit, placed parallel to the ruled lines,

falls on the grating, the spaces A and B become sources of new wavelets which travel out in all directions behind the grating and may, by means of a lens, be brought to a focus on a screen SS. The point P on the screen is equidistant from A and B, and wavelets traveling along the paths AP and BP are in the same phase and form a new wave front at equal distances from A and B which, when brought to a focus at P forms an image of the slit. This is called the central image. For some point R on the screen, the distance AR is greater than the distance BR by an amount AD which is equal to one wave length, so that the wavelets at D and B being in the same phase, form a new wave front. When this is brought to a focus at R it produces an image of the slit called the first order spectrum. Similarly for some point on the screen beyond R a new wave front is formed by a wavelet just starting from B and one from A which has passed over two wave lengths. This produces on the screen the second order spectrum; and so on for higher orders. An exactly similar set of images is formed in the same way on the other side of P. For all points between these images there is interference between the wavelets from A and B, and more especially when the path difference from A and B is an odd multiple of a half wave length, the waves are in opposite phase, and complete interference results in a dark "image" or space on the screen. The same explanation applies in the case of more than two ruled lines, but the spectra in this case are much brighter because of the greater number of lines from which the light comes. If in place of monochromatic light we use white light the central image will be white, because all wave lengths travel over equal distances to this point. But the distance AD will differ with the wave lengths, consequently the first order images will not all be formed at R, but at some point nearby. As a

result a continuous spectrum of all the colors will be formed in the neighborhood of R, with the violet (due to the shortest wave lengths) nearest to the central image. The same is true of the spectra of higher orders.

Let the distance AB between adjacent rulings be a, the difference of path AD be λ, and the angle ABD be θ. Then from triangle ABD we have

$$\lambda = a \sin \theta \qquad (1)$$

or if the angle of the nth order spectrum be ϕ

$$n\lambda = a \sin \phi. \qquad (2)$$

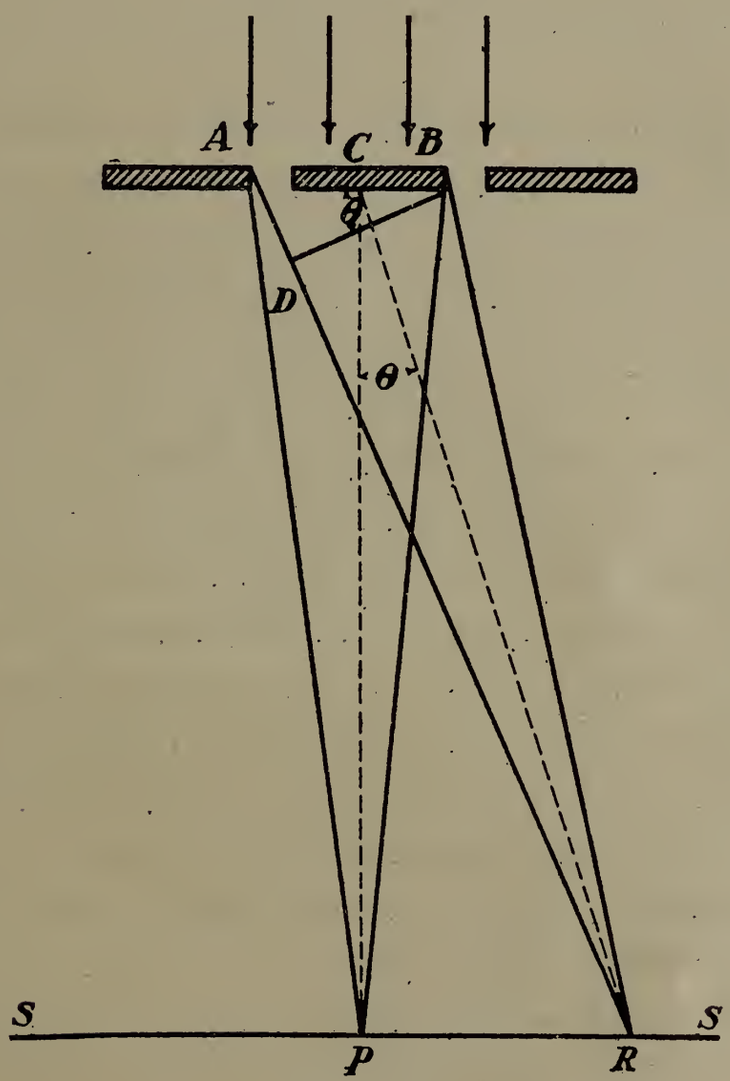


Fig. 70

As the triangles ABD etc., are not exactly right triangles these equations are not quite true, but for the lower orders

of spectra they may be used without making an appreciable error.

If the grating is placed on a spectrometer table, the angle θ may be measured directly. The number of lines per centimeter is usually marked on the grating so that a is easily calculated. From these we calculate the value of λ.

If a spectrometer is not available, we may fix a scale in front of the grating, and move a telescope along it to measure the distance PR between images. The distance RC (or PC, between grating and scale, since these are pactically equal) is readily measured and the sine of θ will be PR/RC.

DIRECTIONS.—Focus the eyepiece of the telescope on the cross hairs and, having focussed the telescope for parallel light, focus the slit of the collimator so that there is no parallax between the image of the slit and the cross hairs of the telescope. (See Exp. 51). Place the grating in the holder on the platform so that the ruled side of the grating is turned away from the collimator. See that the grating lines are vertical and the plane of the grating is at right angles to the axis of the collimator. Make the slit of the collimator very narrow and illuminate it with a sodium flame. Place the cloth over the spectrometer to cut off external light. Move the telescope so as to be in line with the collimator and, having the cross hairs on the image of the slit, observe the readings of the vernier. All parts of the spectrometer, except the telescope, should be clamped.

Move the telescope a few degrees to one side and adjust it till the cross hairs are on the image of the slit as seen in the first order spectrum. The slit must be vertical and parallel to the rulings on the grating for best definition. If the line is not readily found, an incandescent light may be placed in front of the slit which will produce a continuous spectrum

that is readily located and then the sodium light substituted for it. Note the readings of the verniers in degrees, minutes, and seconds of arc, when the cross hairs are on the first order spectrum, and find the mean of the differences between corresponding vernier readings for the first order spectrum and the central image. This is one value of θ. Move the telescope to the other side of the central image and make a second determination of the angle θ by adjusting the telescope so that the image of the first order spectrum on that side is in coincidence with the cross hairs. Repeat each determination two times and find the mean of the six values of θ (three on one side and three on the other side,) the angle through which the telescope must be turned from its position of coincidence with the central image to its position of coincidence with the first order spectra. Note the number of lines to the centimeter as recorded on the grating and determine the distance a between two adjacent lines. Substitute the values of a and θ in equation (1) and calculate λ the wave length of sodium light.

In a similar manner make three determinations of the angle ϕ for coincidence of the cross hairs with the image of the second order spectra on each side of the central image, and find the mean of the six values of ϕ thus observed. The second order spectrum will be at an angle about twice the angle for the first order. It will be rather indistinct, and consequently care must be taken to have the slit well illuminated and all external light cut off the grating as much as possible. Substitute the value of ϕ found for the second order spectra together with the value of a in equation (2) where n has the value 2, and calculate the wave length λ of sodium light.

By careful adjustments it will be possible to see that the sodium line is composed of two lines, and especially so in

the second order spectra. Replace the sodium light by the incandescent lamp and observe the order of the colors in the spectra of both the first and second orders on each side of the central image. Observe which spectra are the most distinct and which spread out the most. Make a sketch in your report showing the order of the colors in diffraction grating spectra.

QUESTIONS.—1.—How does the order of the colors in a grating spectrum differ from the order in a prism spectrum?

2.—What is the frequency of the vibration which produces a yellow sodium line?

EXPERIMENTS IN SOUND.

THEORY AND DEFINITIONS.—Sound is transmitted by means of longitudinal waves, the direction of vibration of the particles of the transmitting medium being parallel to the direction of propagation of the wave. The wave is transmitted as a series of alternate condensations and rarefactions of the medium. The length of the wave is the distance from any one vibrating particle to the next succeeding (or preceding) one which has the same phase. It is therefore the distance between two successive points of maximum condensation or of maximum rarefaction.

The velocity of the wave transmission is the wave length times the frequency of the vibration producing it. The pitch of the sound depends upon the frequency, the intensity upon the amplitude, and the timbre upon the shape of the wave.

When waves impinge upon an elastic body which has the same natural period or the same frequency as the wave, the

body is set in sympathetic vibrations and there is said to be resonance between the two.

When two bodies vibrating near each other have very nearly the same natural period, pulsations of sound or beats, are produced due to the alternate reinforcement and interference of the waves. Two bodies may be brought to the same pitch by altering the period of one of them until the beats disappear and a sound of uniform intensity is heard.

When a column of air in a tube closed at one end, is set in vibration by resonance, there is a condensation at the closed end or a node is formed there and a rarefaction at the open end producing an antinode. The shortest closed tube in which resonance could be produced by a given wave would have a length equal to one-quarter of the wave length. The next length in which resonance would occur would be three-quarters of the wave length, etc. In an open tube resonance would occur for lengths equal to multiples of a half wave length.

When a stretched string is set in vibration, the wave length of the sound produced cannot be greater than twice the length of the string since there must be a node at each end.

It may be shown that the frequency of vibration or the pitch of the sound produced is inversely proportional to the length of string and to the square root of its mass per unit length, but directly proportional to the square root of its tension. If T represents tension, l, length of string, and m, mass per unit length, the frequency is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

53. GRAPHICAL DETERMINATION OF PITCH OF TUNING FORKS.

To determine the pitch of two tuning forks graphically. (D. 592, 609; G. 245; K. 304; W. 267, 288.)

APPARATUS.—Two tuning forks to be tested, a long board slide-way with slides and clamps for pendulum and tuning fork, strip of plate glass, rubber hammer for fork, steel dividers, paste of whiting or chalk dust and dauber.

DESCRIPTION AND METHOD.—The slide-way is a smooth board about 5 feet long and 6 inches wide, provided with side guard strips. The wooden slide—about 2 feet long—slips easily in the slide-way. A tuning fork and a small pendulum are supported above the slide-way so that they may vibrate across the path of the slider. The clamps allow an adjustment of the pendulum and the fork. The strip of glass may be attached to the top of the slide by appropriate fasteners.

The method used is to get traces of the vibrating fork and swinging pendulum in the same line on the glass and to count the number of waves of the fork in one vibration or wave length of the pendulum; then by timing the pendulum vibration, to determine the number of vibrations of the fork in one second, *i. e.*, its pitch, by dividing the number of oscillations of the fork corresponding to one oscillation of the pendulum by the time required for the latter.

DIRECTIONS.—Take the wooden slide and attached strip of glass to the sink and clean the glass thoroughly with a dry cloth. Cover the glass with a uniform coating of whiting paste and let it dry. Clamp the pendulum and the tuning fork in place, so that the vibrating end of the fork is near the pendulum. Fasten a stylus of copper foil at the end of

each, so that if the slide were pushed through, the lines traced would coincide approximately. It is best to have the styluses touch the glass lightly and slant in the same direction that the slide is to pass. Move the slide back until the styluses are free to vibrate, then set the pendulum vibrating through a small arc, and strike the fork a single sharp blow. Pass the slides through at a moderate rate so that the pendulum traces one or more complete oscillations. Count the number of oscillations of the fork in a complete oscillation of the pendulum. In order to compare the fork trace with the pendulum trace made at the same time, it is necessary to shift the crossing point of the pendulum trace the distance between the two styluses. This is because one stylus makes its trace on the glass before the other touches the glass. See Fig. 71 where the dotted line shows the shifted position for the pendulum trace. Repeat the experiment until two values of the fork oscillations are found that differ by two or less. Repeat for the second fork. Observe the time for 100 complete oscillations of the pendulum, and calculate the time for one oscillation. Repeat as a check. Calculate the number of oscillations for each fork in one second.

Question —Describe exactly how your results are affected by the friction of the styluses on the glass.

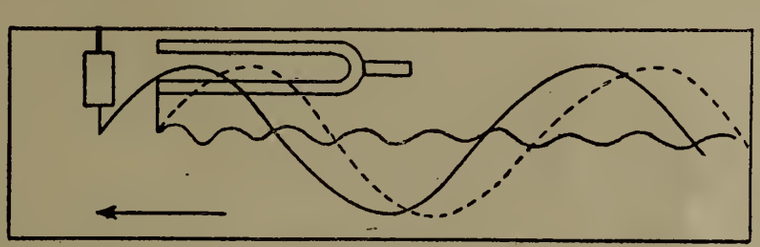


Fig. 71

54. VIBRATIONS OF STRETCHED STRINGS.

To test the relation between frequency of vibration of a stretched string and its length, tension and mass. (D. 602; G. 266-270; K. 326, 327; W. 267, 276.)

APPARATUS.—Sonometer box, two tuning forks, two steel wires of different cross sections, two masses for stretching wires.

DESCRIPTION.—The sonometer box is a light pine box about 4 feet long, 3 inches high, and 4 inches broad. See Fig. 72. It is used to strengthen the feeble sound of the wire. The box rests on rubber feet to prevent the communication of the vibrations to the table. On top of the box is a fixed fret and a moveable one, also a quarter circle of brass that rocks in

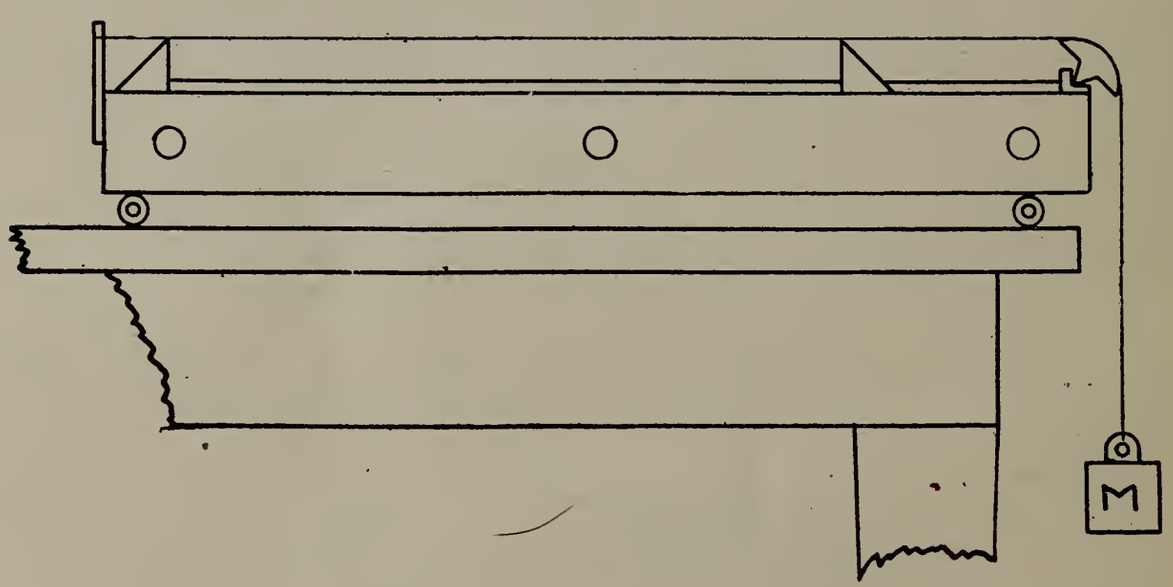


Fig. 72

a V-groove of the same metal at the end of the box. The wire is fastened to a brass post at one end of the box, and passes over both frets and the small groove of the quarter circle. The stretching mass is hung from a ring at the end of the wire.

Directions.—*Relation between pitch of wire and length.*—Use the smaller wire and hang the larger iron block (about 4,000 gms.) to the free end of the wire. See that the quarter circle rocks freely in the groove so that the suspended mass gives the correct tension of the wire. Strike the fork a single sharp blow with the rubber hammer, place its stem on the sounding box, lightly press the wire down on the moveable fret and pluck the wire between the frets. Alter the position of the moveable fret until the note of the string is nearly in unison with the fork. If the student has not a musical ear, he can find this position of approximate unison by placing a small rider of paper on the wire and (without plucking the wire) moving the free fret back and forth until a position is found where the rider is thrown off violently. The wire is now nearly the length necessary for unison of pitch with the fork. Strike the fork again, touch it to the box and move the fret backward or forward a millimeter at a time noting the forced vibrations of the wire. A position will be found where the "swell" of the wire is a maximum. Note this length of the wire as the length for unison with the fork. The swell is more noticeable when the vibration of the fork is vigorous, hence the fork should be struck and tried frequently. A distance of a millimeter to either side of the maximum swell length will cause the wire to pulsate (the phenomenon of beats) instead of vibrating in a single prolonged swell.

By this same process determine the length of wire in unison with the second fork. Record the numbers of the forks and their pitches, also record the lengths of wire that vibrate in unison with them. Test with the data found the relation that pitch varies inversely with length.

Relation between pitch and tension.—Determine as before the length of the smaller wire in unison with the first fork

having the larger mass (about 4,000 gms.) hung to the free end of the wire. Determine by the law just proved the pitch for 100 cm. length of the wire. Repeat using same fork and both iron blocks (about 6,000 gms.). Calculate the pitch P for 100 cm. length. Having thus the pitches for 100 cm. lengths of wire for tension of 4,000 and 6,000 gms., test the relation that pitch varies directly with square root of the tension.

Relation between pitch and size of wire.—Using small wire and tension of about 6,000 gms., determine the length in unison with the first fork and calculate the pitch P for a length of 100 cm. Repeat using the same fork and tension, but the larger wire. Calculate the pitch of 100 cm. length. Note on the small card attached to wires the mass per cm., then test the relation that pitch varies inversely as the square root of mass per cm. of length.

QUESTION.—In what way do your results depend upon the temperature at which the experiment was performed?

55. RESONANT AIR COLUMNS.

(*a*). To determine the velocity of sound in air by resonant air columns. (*b*). To determine the pitch of a tuning fork. (*c*). To determine the correction factor due to the disturbance at the open end of the tube. (D. 382-383; G. 278-279; K. 308-309; W. 316.

APPARATUS.—Resonance apparatus, two tuning forks, rubber hammer.

DESCRIPTION AND THEORY.—If a tuning fork is vibrated over a column of air its vibrations are reinforced and made audible to the ear providing the column of air is of the proper

length to produce resonance. Neglecting the correction due to the disturbance at the open end of the tube, resonance first occurs when the length of the confined air column is equal to one-fourth of the length of the waves set up by the fork used.

Resonance will again occur when the length of the air column is further increased by exactly half a wave length.

Let λ denote the wave length of the sound produced by the fork, l_1, the length of the air column for first resonance and l_2 the length for second resonance, each measured from the mouth of the tube; then for first resonance

$$\lambda = 4(l_1 + c) \tag{1}$$

where c is the correction factor due to the disturbance at the open end of the tube. For second resonance.

$$\lambda = 2(l_2 - l_1) \tag{2}$$

The velocity of sound is equal to the number of vibrations per second times the wave length, or

$$v = n\lambda \tag{3}$$

Combining (2) and (3) gives for the value of v

$$v = 2n(l_2 - l_1) \tag{4}$$

Equating the right members of equations (1) and (2) gives for the value of the correction factor c

$$c = \frac{l_2 - 3l_1}{2} \quad . \quad . \quad . \quad . \quad . \quad 5)$$

According to Lord Rayleigh this correction should be about equal to the radius of the tube. The velocity of sound in air in meters per second is given approximately by

$$V_t = 331.7 + .6t \tag{6}$$

where t is the temperature of the air.

The resonance apparatus consists of a glass tube T, mounted vertically in front of a scale S, Fig. 73, with a tuning

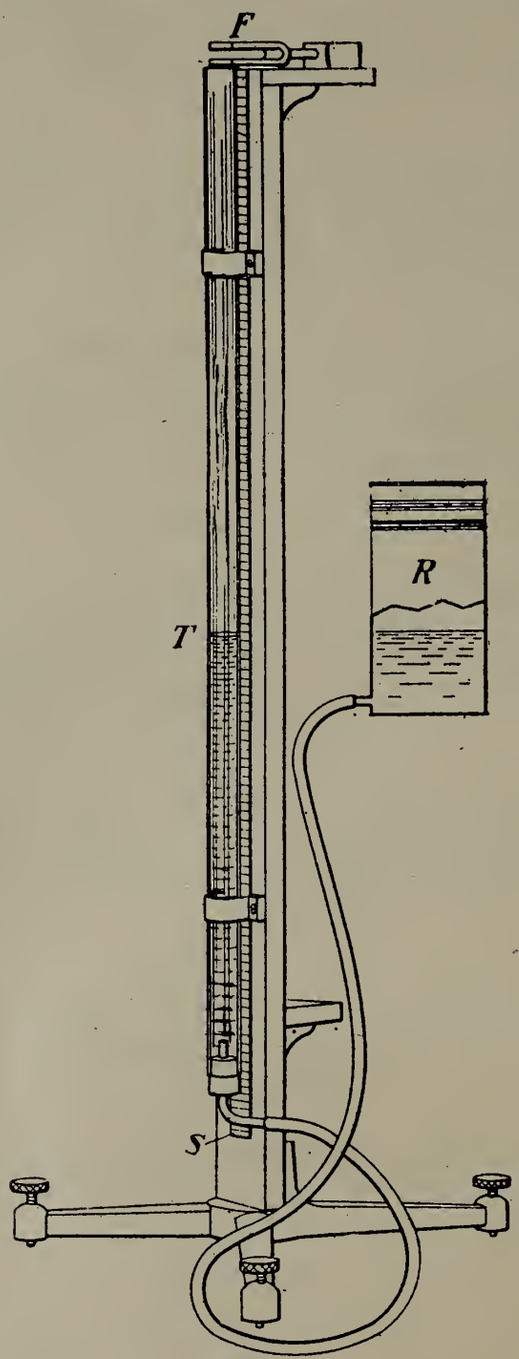


Fig. 73

fork F clamped above the open end of the tube. The water level in the tube can be varied by raising or lowering the reservoir R.

DIRECTIONS.—Clamp one of the tuning forks (the one whose frequency is given) in position over the tube as shown

in Fig. 73. Strike the fork with the rubber hammer and raise and lower the water level in the tube till the point is found where the tone is reinforced most strongly. Measure the distance from this water level to the top of the tube and, if it is the first point of resonance, call it l_1. Avoid parallax when taking a reading. Six observations should be taken for the position of each node, part of them with the water rising and part with it lowering.

Lower the water level until the second point of resonance is obtained. Determine this in a manner similar to the first and designate it as l_2. Record the number of the fork, its frequency and the temperature of the room. Substitute in equation (4) and determine the velocity v. Calculate the theoretical value from equation (6). Determine the percent error in your results. Also determine the correction factor c from equation (5).

Clamp the second fork (frequency unknown) in position and determine the two points of resonance as described above. Substitute in equation (4) using for v the value obtained from equation (6) and solve for n.

Questions.—(1) Show by diagram that l_1 is one-quarter of a wave length and that l_2 is three-quarters wave length.

(2)—What is the ratio of the correction factor c to the radius of the tube?

Appendix.

I. COMMON CONVERSION FACTORS.

Mass

1 gram = 15.432 grains	1 grain = 0.0648 gram.
1 gram = 0.0352 oz. av.	1 oz. av. = 28.35 gram.
1 kg. = 2.204 lbs.	1 lb. = 0.4536 kg.

Length

1 centimeter = 0.3937 inch	1 inch = 2.539 cms.
1 meter = 3.280 feet.	1 foot = 0.3047 meter.
1 kilometer = 0.6213	1 mile = 1.609 km.
1 micron = 0.001 m m.	1 mile = 0.001 inch

Area.

1 sq. cm. = 0.1550 sq. in.	1 sq. in. = 6.451 sq. cm.
1 sq. m. = 10.76 sq. ft.	1 sq. ft. = 0.0929 sq. m.

Volume.

1 cu. cm. = 0.0610 cu. in.	1 cu. in. = 16.38 cu. cm.
1 cu. m. = 35.31 cu. ft.	1 cu. ft. = 0.02832 cu. m.
1 liter = 1.760 pints	1 quart = 1.135 liters.

Angles.

1 radian = 57.295 degrees	1 degree = 0.01745 radian.

Temperature.

1° C. = (9/5)°F.	1° F. = (5/9)° C.

II. SPECIFIC GRAVITY.

Air, at 0° and 1 atmo.,	.001293	Iron, wrought	7.8
Alcohol, at 20	.789	Kerosene	.79
Aluminum	2.6	Lead	11.37
Brass	8.5	Mercury, at 20°	13.55
Copper	8.9	Platinum	21.50
Ether, at 20°	.715	Quartz	2.65
German Silver	8.5	Silver	10.53
Glass, common	2.4—2.6	Steam, at 100°	.00058
Glass, flint	3. —5.9	Steel	7.8
Hydrogen, at 0° and 1 atmo	.0000899	Sulphuric acid	1.832
Ice	.917	Turpentine	.87
Iron, cast	7.1—7.7	Wood's metal	9.7
		Wood, oak	.6—.9
		Wood, pine	.4—.7
		Wood, walnut	.6—.8

III. ELASTIC CONSTANTS.

	Modulus of Elasticity or Young's Modulus.		Modulus of Rigidity or Simple Rigidity.	
	dynes/sq.cm.	lbs./sq. in.	dynes/sq.cm	lbs./sq. in.
Multiply by	10^{11}	10^{6}	10^{11}	10^{6}
Brass:				
cast	6.5	9	2.4	3.5
wire	10	14	3.7	5.4
Copper:				
annealed	10	14	—	—
cast	12	17	4.	6.
wire	12	17	4.5	6.5
Iron:				
annealed	21	30		
cast	12	17	5.3	7.6
wire	19	26	8.0	12
wrought	20	28	7.7	11
Steel:				
Bessemer	22	31		
cast	20	28	8.0	12
wire	19	26		

IV. MOMENTS OF INERTIA

(m=mass of body, l=length, b=breadth. r=external radius, r_1= internal radius.)

Body	Axis	Moment of Inertia
Circular disk or cylinder	perpendicular through center of flat face	$\frac{1}{2}\, m\, r^2$
Ring or hollow cylinder	perpendicular through center of flat face	$\frac{1}{2}\, m\, (r^2+r_1^2)$
Circ. cylinder	transverse through center	$m(r^2/4+l^2/12)$
Thin rod	transverse through end	$ml^2/3$
Thin rod	transverse through middle	$ml^2/12$
Rectangular block	through center perpendicular to face with sides l and b	$m(l^2+b^2)/12$
Sphere	through center	$2mr^2/5$
Hollow sphere	through center	$\frac{2}{5}\, m\, \frac{(r^5-r_1^5)}{(r^3-r_1^3)}$

V. BAROMETRIC CORRECTIONS.

Capillary Correction=+0.031 cm.

Temperature Corrections

Temp. °F.	Correction.	Temp. °F.	Correction.
60	0.171 cm.	70	0.225 cm.
61	0.176 cm.	71	0.231 cm.
62	0.181 cm.	72	0.236 cm.
63	0.185 cm.	73	0.241 cm.
64	0.192 cm.	74	0.246 cm.
65	0.198 cm.	75	0.251 cm.
66	0.203 cm.	76	0.256 cm.
67	0.209 cm.	77	0.261 cm.
68	0.214 cm.	78	0.266 cm.
69	0.220 cm.	79	0.271 cm.

NOTE—Temperature readings should be taken from the Fahrenheit thermometer on the side of the barometer.

VI. HEAT CONSTANTS.

Solids

Substance	Coefficient of Linear Expansion	Specific Heat
Aluminum	.0000255	.219
Brass	.0000189	.090
Copper	.0000169	.0936
Glass (crown)	.0000078	.161
Glass (flint)	.000009	.117
German Silver	.000018	.095
Ice	.0000507	.502
Iron (cast)	.0000102	.11
Iron (wrought)	.0000119	.11
Lead	.0000276	.031
Nickel	.0000128	.109
Platinum	.0000089	.032
Quartz (fused)	.00000039	.174
Silver	.0000188	.056
Steel (tempered)	.0000132	.12
Tin	.0000214	.055
Zinc	.0000263	.094

Liquids, Vapors and Gases

(at constant pressure).

Substance	Coefficient of Cubical Expansion	Specific Heat
Air	.003671	.237
Alcohol (ethyl)	.00110	.548
Alcohol Vapor	.	.453
Benzine	.00124	.40
Benzine Vapor		.299
Ether (ethyl)	.00163	.54
Hydrogen	.003661	3.409
Kerosene	.0012	.39
Mercury	.0001818	.0333
Sulphuric Acid	.00059	.34
Turpentine	.00094	.420
Water (15°—100°)	.000372	1.
Water Vapor	.	.442

VII. STEAM TEMPERATURE.

At Different Barometric Pressures.

h.cm.	70	71	72	73	74	75	76	77
.0	97.72	98.11	98.50	98.88	99.26	99.63	100.00	100.36
.1	97.76	98.15	98.54	98.92	99.30	99.67	100.04	100.40
.2	97.80	98.19	98.58	98.96	99.33	99.71	100.07	100.44
.3	97.84	98.23	98.61	98.99	99.37	99.74	100.11	100.47
.4	97.88	98.27	98.65	99.03	99.41	99.78	100.15	100.51
.5	97.92	98.31	98.69	99.07	99.44	99.82	100.18	100.55
.6	97.96	98.34	98.73	99.11	99.48	99.85	100.22	100.58
.7	98.00	98.38	98.77	99.14	99.52	99.89	100.26	100.62
.8	98.03	98.42	98.80	99.18	99.56	99.93	100.29	100.65
.9	98.07	98.46	98.84	99.22	99.59	99.96	100.33	100.69

VIII. HYGROMETRY

Pressure of aqueous vapor, p, and weight of water, m, contained in one cubic meter of air **saturated** at the temperature t°C.

t	p	m	t	p	m	t	p	m
	cm.	gm.		cm.	gm.		cm.	gm.
—10	.22	2.4	4	.61	6.4	18	1.54	15.2
— 9	.23	2.5	5	.65	6.8	19	1.64	16.2
— 8	.25	2.7	6	.70	7.2	20	1.74	17.1
— 7	.27	2.9	7	.75	7.7	21	1.85	18.2
— 6	.29	3.1	8	.80	8.2	22	1.97	19.3
— 5	.32	3.4	9	.86	8.8	23	2.09	20.4
— 4	.34	3.7	10	.92	9.4	24	2.22	21.6
— 3	.37	4.0	11	.98	10.0	25	2.35	22.8
— 2	.40	4.2	12	1.05	10.6	26	2.50	24.1
— 1	.43	4.5	13	1.12	11.3	27	2.65	25.5
0	.46	4.9	14	1.19	12.0	28	2.81	27.0
1	.49	5.2	15	1.27	12.7	29	2.98	28.5
2	.53	5.6	16	1.36	13.5	30	3.16	30.1
3	.57	6.0	17	1.45	14.4	31	3.34	31.7

IX. SPECIFIC RESISTANCE.

(Ohms per cm.^{3} at 0°C.)

Aluminum	$3. \times 10^{-6}$	Manganin	$42. \times 10^{-6}$
Brass	8.	Mercury	94.
Copper	1.5	Nickel	10.
German Silver	20.	Platinum	8.9
Gold	2.	Silver	1.5
Iron	10.5	Steel	50.
Lead	20.	Zinc	5.7

X. ELECTROCHEMICAL EQUIVALENTS.

Element	Atomic Weight	Valency	Chemical equivalent	Electrochemical equivalent
Chlorine	35.45	1	35.45	.0003672
Copper, (cupric)	63.6	2	31.8	.000329
Hydrogen	1.008	1	1.008	.00001044
Iron, (ferric)	55.9	2	27.95	.000289
Iron, (ferrous)	55.9	3	18.49	.000193
Oxygen	16.0	2	8.0	.00008283
Silver	107.93	1	107.93	.001118
Zinc	65.4	2	32.7	.000338
Nickel	58.6	2	29.3	.000305

XI. INDEX OF REFRACTION FOR D LINE

(λ=0.00005893)

Air	1.000294	Cedar oil	1.516
Alcohol, ethyl	1.362	Ether	1.36
Alcohol, methyl	1.332	Glass (crown)	1.51–1.62
Benzine	1.504	Glass (flint)	1.57–1.96
Canada Balsam	1.54	Quartz	1.54
Carbon bisulphide	1.63	Water	1.334

Notes on Tables XII and XIII

Table XII—The functions of angles between 0 and 45° (left hand column) are marked at top of page, those from 45° to 90° (right hand column) at bottom. For decimal fractions of a degree, add proportional parts of the difference given in columns next to functions.

Table XIII—Columns on right of page give numbers to be added to logarithms for the corresponding decimal fractions given at the head of these columns.

XII. NATURAL TRIGONOMETRICAL FUNCTIONS

Degree	Sine		Tangent		Cotangent		Cosine		Degree
0	0.0000		0.0000		∞		1.0000		90
1	.0175	175	.0175	175	57.29		0.9998	02	89
2	.0349	174	.0349	174	28.64		.9994	04	88
3	.0523	174	.0524	175	19.08		.9986	08	87
4	.0698	175	.0699	175	14.30		.9976	10	86
		174		176				14	
5	0.0872		0.0875		11.43		0.9962		85
6	.1045	173	.1051	176	9.514		.9945	17	84
7	.1219	174	.1228	177	8.144		.9925	20	83
8	.1392	173	.1405	177	7.115		.9903	22	82
9	.1564	172	.1584	179	6.314	801	.9877	26	81
		172		179		643		29	
10	0.1736		0.1763		5.671		0.9848		80
11	.1908	172	.1944	181	5.145	526	.9816	32	79
12	.2079	171	.2126	182	4.705	440	.9781	35	78
13	.2250	171	.2309	183	4.331	374	.9744	37	77
14	.2419	169	.2493	184	4.011	320	.9703	41	76
		169		186		279		44	
15	0.2588		0.2679		3.732		0.9659		75
16	.2756	168	.2867	188	3.487	245	.9613	46	74
17	.2924	168	.3057	190	3.271	216	.9563	50	73
18	.3090	166	.3249	192	3.078	193	.9511	52	72
19	.3256	166	.3443	194	2.904	174	.9455	56	71
		164		197		157		58	
20	0.3420		0.3640		2.747		0.9397		70
21	.3584	164	.3839	199	2.605	142	.9336	61	69
22	.3746	162	.4040	201	2.475	130	.9272	64	68
23	.3907	161	.4245	205	2.356	119	.9205	67	67
24	.4067	160	.4452	207	2.246	110	.9135	70	66
		159		211		101		72	
25	0.4226		0.4663		2.145		0.9063		65
26	.4384	158	.4877	214	2.050	95	.8988	75	64
27	.4540	156	.5095	218	1.963	87	.8910	78	63
28	.4695	155	.5317	222	1.881	82	.8829	81	62
29	.4848	153	.5543	336	1.804	77	.8746	83	61
		152		231		72		86	
30	0.5000		0.5774		1.732		0.8660		60
31	.5150	150	.6009	235	1.664	68	.8572	88	59
32	.5299	149	.6249	240	1.600	64	.8480	92	58
33	.5446	147	.6494	245	1.540	60	.8387	93	57
34	.5592	146	.6745	251	1.483	57	.8290	97	56
		144		257		55		98	
35	0.5736		0.7002		1.428		0.8192		55
36	.5878	142	.7265	263	1.376	52	.8090	102	54
37	.6018	140	.7536	271	1.327	49	.7986	104	53
38	.6157	139	.7813	277	1.280	47	.7880	106	52
39	.6293	136	.8098	285	1.235	45	.7771	109	51
		135		293		43		111	
40	0.6428		0.8391		1.192		0.7660		50
41	.6561	133	.8693	302	1.150	42	.7547	113	49
42	.6691	130	.9004	311	1.111	39	.7431	116	48
43	.6820	129	.9325	321	1.072	39	.7314	117	47
44	.6947	127	.9657	332	1.036	36	.7193	121	46
		124		343		36		122	
45	0.7071		1.0000		1.000		0.7071		45
Degree	Cosine		Cotangent		Tangent		Sine		Degree

XIII. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Logarithms—Con.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8219	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9235	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9566	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

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